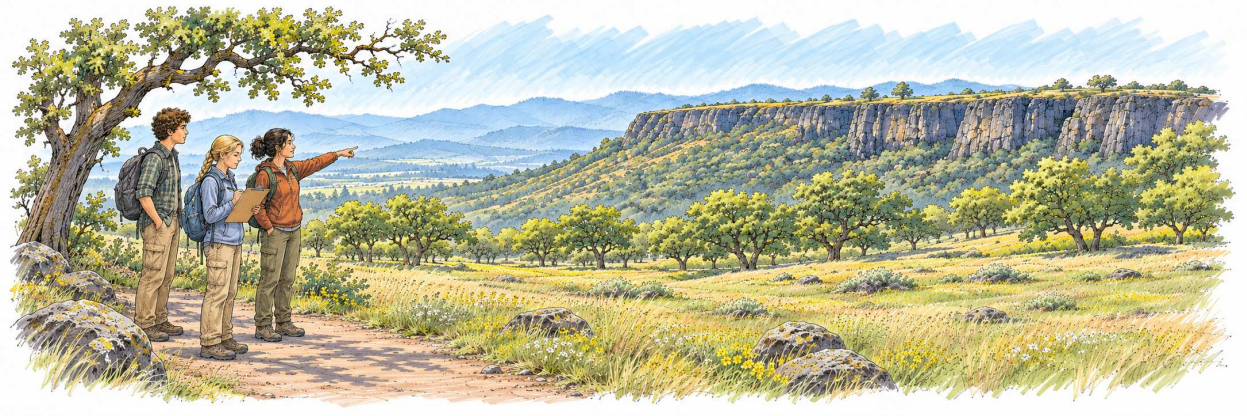
SAINT MARY'S SCHOOL • MEDFORD, OREGON

AP Statistics

Investigations and Workbook

Guided investigations, independent practice, exit checks, and nightly homework for each lesson. Use the workbook to test a claim, record evidence, and explain a conclusion in context.



01 02 03 04 05 06 07 08 09 10

Materials customized by **Jason Javarone** for use in the St. Mary's School AP Statistics course.

Source-based classroom edition | November 11, 2026

Ten SMS units follow the five-unit AP Statistics Course and Exam Description, effective Fall 2026.

About this edition

For each statistical conclusion, name the group and the quantity, give numerical evidence with units when appropriate, and explain what the evidence supports. Read your answer with the question covered. Someone who has never seen the prompt should still understand your claim.

Match the lesson ID across books. For example, S01.01 in the Student Reference pairs with S01.01 in the Workbook. The homework page gives the next reading number.

The primary instructional source is Introductory Statistics 2e by Barbara Illowsky and Susan Dean, OpenStax, Rice University (supplied 2026 printing of the second edition). The explanations and credited worked examples adapt this textbook to the AP course. Source notes identify the original sections and problems and any changes to context, assumptions, or calculations. Local applications and class-data investigations provide additional practice.

This adaptation is distributed under Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0), consistent with the supplied OpenStax edition. The original OpenStax textbook is available in full for free at OpenStax.org: https://openstax.org/details/books/introductory-statistics-2e

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Local artwork was created for this SMS edition. Chapel architecture follows the supplied Ron Grimes/Architects reference; the historical academy follows a supplied archival photograph credited to the Southern Oregon Historical Society. Two approved Table Rock landscapes and the supplied SMS logo are reused unchanged. Captions distinguish local settings from observed data. Original reference art and the school logo retain their original ownership.

Print US Letter, actual size, long-edge duplex. The inner margin is 1 inch; the outer margin is 0.75 inch. Color assists navigation, but labels carry the meaning. Fonts are embedded Caladea and Carlito. Keep teacher solutions separate from student copies.

Contents

About this edition 2

Visual guide and navigation 7

Working with evidence 7

Unit 01 | Analyzing Data 8

S01.01 | Questions, Variables, and Categorical Data . . . 9
S01.01-HW | Nightly practice . . . 11
S01.02 | Two Categorical Variables and Association . . . 13
S01.02 | Optional lab | Do the two bags have the same mix? . . . 15
S01.02-HW | Nightly practice . . . 17
S01.03 | Seeing a Quantitative Distribution . . . 19
S01.03-HW | Nightly practice . . . 23
S01.04 | Center, Variability, and Resistance . . . 25
S01.04-HW | Nightly practice . . . 27
S01.05 | Quartiles, Outliers, and Group Comparisons . . . 29
S01.05-HW | Nightly practice . . . 31
S01.06 | Relative Standing and Changing Units . . . 33
S01.06-HW | Nightly practice . . . 35
Unit 1 | Cumulative synthesis . . . 37

Unit 02 | Collecting Data 38

S02.01 | Choosing a Random Sample . . . 39
S02.01-HW | Nightly practice . . . 41
S02.02 | Bias, Observational Studies, and Ethical Questions . . . 43
S02.02-HW | Nightly practice . . . 45
S02.03 | Designing a Completely Randomized Experiment . . . 47
S02.03-HW | Nightly practice . . . 49
S02.04 | Blocking, Matched Pairs, and Scope of Inference . . . 51
S02.04-HW | Nightly practice . . . 55
Unit 2 | Cumulative synthesis . . . 57

P01 | How much can a sample change the story? 58

Unit 03 | Probability 60

S03.01 | Chance models and reproducible simulation . . . 61
S03.01-HW | Nightly practice . . . 63
S03.02 | Events, complements, and overlapping categories . . . 65
S03.02 | Optional lab | Doubles or a sum of eight . . . 67
S03.02-HW | Nightly practice . . . 69
S03.03 | Conditional probability and independence . . . 71
S03.03-HW | Nightly practice . . . 75
S03.04 | Multi-stage chance processes and probability trees . . . 77
S03.04-HW | Nightly practice . . . 81
Unit 3 | Cumulative synthesis . . . 83

Unit 04 | Random Variables 84

S04.01 | Random variables, distributions, and expected value . . . 85
S04.01-HW | Nightly practice . . . 87
S04.02 | The binomial model . . . 89
S04.02-HW | Nightly practice . . . 93
S04.03 | Continuous models and the normal distribution . . . 95
S04.03-HW | Nightly practice . . . 97
S04.04 | Normal probabilities, percentiles, and cutoffs . . . 99
S04.04-HW | Nightly practice . . . 103
Unit 4 | Cumulative synthesis . . . 105

Unit 05 | Sampling Distributions 106

S05.01 | Sampling distributions and randomization distributions . . . 107
S05.01-HW | Nightly practice . . . 111
S05.02 | Sampling distributions of proportions and their differences . . . 113
S05.02 | Optional lab | Bead draws | How much can a sample proportion vary? . . . 115
S05.02-HW | Nightly practice . . . 117
S05.03 | Sample means, differences in means, and the CLT . . . 119
S05.03-HW | Nightly practice . . . 121
Unit 5 | Cumulative synthesis . . . 123

Unit 06 | Confidence Intervals for Proportions 124

S06.01 | What a confidence interval says . . . 125
S06.01-HW | Nightly practice . . . 129
S06.02 | Build and justify a one-proportion interval . . . 131
S06.02 | Optional lab | Globe toss lab . . . 133
S06.02-HW | Nightly practice . . . 137
S06.03 | Plan precision before collecting data . . . 139
S06.03-HW | Nightly practice . . . 141
S06.04 | Estimate a difference in population proportions . . . 143
S06.04-HW | Nightly practice . . . 147
Unit 6 | Cumulative synthesis . . . 149

Unit 07 | Hypothesis Tests for Proportions 150

S07.01 | Hypotheses, null models, and p-values . . . 151
S07.01-HW | Nightly practice . . . 153
S07.02 | Carry out a one-proportion z test . . . 155
S07.02-HW | Nightly practice . . . 159
S07.03 | Errors, power, and consequences . . . 161
S07.03-HW | Nightly practice . . . 163
S07.04 | Test a difference in population proportions . . . 165
S07.04-HW | Nightly practice . . . 169
S07.05 | Compare categorical distributions with chi-square . . . 171
S07.05-HW | Nightly practice . . . 173
S07.06 | Investigate association with chi-square . . . 175
S07.06-HW | Nightly practice . . . 179
Unit 7 | Cumulative synthesis . . . 181

Unit 08 | Confidence Intervals for Means 182

S08.01 | Estimating a population mean . . . 183
S08.01-HW | Nightly practice . . . 185
S08.02 | Confidence, precision, and defensible claims . . . 187
S08.02-HW | Nightly practice . . . 189
S08.03 | Paired observations: analyze the differences . . . 191
S08.03 | Optional lab | Does the launch surface change a frog's jump? . . . 193
S08.03-HW | Nightly practice . . . 195
S08.04 | Estimating a difference between independent means . . . 197

S08.04 | Optional lab | Which package has more visible chocolate patches per cookie? . . . 198
S08.04-HW | Nightly practice . . . 201
Unit 8 | Cumulative synthesis . . . 203

Unit 09 | Hypothesis Tests for Means **204**

S09.01 | Testing a claim about one population mean . . . 205
S09.01-HW | Nightly practice . . . 209
S09.02 | Testing a population mean difference from paired data . . . 211
S09.02-HW | Nightly practice . . . 215
S09.03 | Testing a difference between independent means . . . 217
S09.03-HW | Nightly practice . . . 219
S09.04 | Choosing inference and linking tests with intervals . . . 221
S09.04-HW | Nightly practice . . . 225
Unit 9 | Cumulative synthesis . . . 227

P02 | Does a design change increase flight time? **228**

Unit 10 | Two-Variable Data and Regression **230**

S10.01 | Seeing relationships in paired quantitative data . . . 231
S10.01-HW | Nightly practice . . . 233
S10.02 | Correlation: strength of a linear association . . . 235
S10.02-HW | Nightly practice . . . 237
S10.03 | Linear predictions and their limits . . . 239
S10.03-HW | Nightly practice . . . 241
S10.04 | Residuals and checking a linear model . . . 243
S10.04-HW | Nightly practice . . . 245
S10.05 | Least squares and explained variation . . . 247
S10.05-HW | Nightly practice . . . 249
Unit 10 | Cumulative synthesis . . . 251

Appendix | AP Statistics formula sheet **253**

Visual guide and navigation

Unit numbers, titles, colors, and page-edge tabs identify your place in the course. Lesson IDs match across the reference, workbook, and teacher guide. Read every graph with its labels, units, and caption; color is a second cue, not the only cue.

01 02 03 04 05 06 07 08 09 10

KEY IDEA

A definition or principle to retain and use in a new situation.

WATCH FOR

A common error, a condition that needs checking, or a conclusion that the evidence does not support.

CONCEPT → WORKED EXAMPLE → YOUR INVESTIGATION

Instructional graphics make relationships visible: compare groups, follow a probability path, locate a statistic in a distribution, or connect a calculation to its meaning. Workbook organizers leave the reasoning for you to complete. Occasional context illustrations introduce a setting; they are not measured data.

A good statistical graphic helps answer a question. Identify what is shown, describe the pattern with evidence, and state what the display cannot establish.

Working with evidence

After the entry ticket, follow the explanation and worked example in the Student Reference. Start the guided application with your own first step, then compare approaches. On pages labeled Opening prediction, make that brief prediction before the lesson explanation. In lessons with an optional lab, choose one guided task, then turn to the common independent questions. Revise an answer and complete the exit check on your own.

Each homework sheet identifies the required problems. Allow 30-40 minutes for practice. Read the passage for the next meeting in Preparation Readings and answer its two checks, allowing 12-15 minutes for both. Other workbook problems are optional.

The next entry ticket asks about the previous lesson and the reading. After a project, checkpoint or review, find your practice in Preparation Readings. Record your time and flag a problem that runs long. The course target averages 45-60 minutes of nightly work.

Unit 01 | Analyzing Data

Statistics begins with a question, a defined population, and measurements that can answer the question. This unit develops the habit of examining a distribution before selecting a numerical summary. You will distinguish categorical and quantitative variables, compare groups with appropriate denominators, and connect every calculation to the original context. Displays and summaries can reveal patterns, but they can also conceal important features. You will learn to identify those limits and to explain what the data support. The local examples use illustrative data; they are not claims about actual Saint Mary's students or Southern Oregon residents.

CED topics: 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.1, 2.2

IN THIS UNIT

Lesson	Focus
S01.01	Questions, Variables, and Categorical Data
S01.02	Two Categorical Variables and Association
S01.03	Seeing a Quantitative Distribution
S01.04	Center, Variability, and Resistance
S01.05	Quartiles, Outliers, and Group Comparisons
S01.06	Relative Standing and Changing Units

Reading an arrival record

Record	Observed value
Case	One student arriving Monday morning
Variable 1	Travel mode: bicycle
Variable 2	Travel time: 14 minutes

A row for each trip is a different record design from a row for each student.

LOCAL QUESTION

At SMS, what would you record to describe how students arrive at school? Define the population, observational unit, and variables before collecting data.

S01.01 | Questions, Variables, and Categorical Data

CED 1.1, 1.2, 1.3, 1.4

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

What does one row represent?

Work with a partner using this illustrative file description: 12 students have a student ID, usual arrival method, number of siblings, and travel time in minutes. Arrival responses are car, bus, car, walk, car, bus, bicycle, car, walk, bus, car, car. No actual student records are needed.

Materials. Paper or a spreadsheet; colored pencils optional.

Build a data dictionary

Observational unit: _____

Variable name	How recorded?	Type	Units / categories

Population / sample: _____

Define one row of the data set before deciding how to summarize its columns.

1. Write one useful investigative question that the file could answer descriptively and one question it could not answer.

2. Complete the data dictionary above for student ID, usual arrival method, number of siblings, and travel time. Identify the observational unit and explain why student ID is categorical even when it contains digits.

3. Build an arrival-method frequency table and relative-frequency table.

Draw your table here. Include headings and all rows needed for the data.

01

S01.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Draw a bar chart. Exchange charts and identify any missing labels or misleading scales.

5. Write two sentences: one numerical description of this group and one limit on generalization.

Discuss your results

- Why is a mean student ID not a useful summary?
- Would recording the 12 students constitute a census? Explain what must be specified.

INDEPENDENT APPLICATION

1. A library selects a random sample of 60 books from its collection of 6,000 books. The mean age of the sampled books is 8.4 years. Identify the observational unit, population, sample, variable, sample statistic, and population parameter of interest.

2. A poll of 50 respondents records 20 first choices for A, 18 for B, and 12 for C. Give percentages and explain why a bar chart is suitable.

3. A school asks students to choose every club they attend. Club percentages sum to 146%. Is this necessarily a mistake? Explain your reasoning.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A survey records the ZIP code and daily water use in liters for each household. Classify each variable and identify the observational unit.

S01.01-HW | Nightly practice

Name _____ Date _____

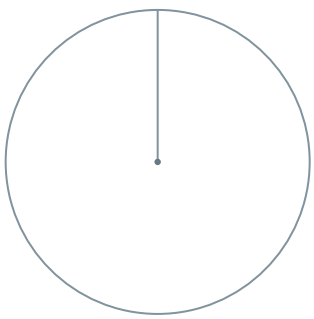
Required problems. 1, 2, 3, 5, 6. **Optional practice.** 4. **Prepare for the next meeting.** Read PR02 for S01.02 and answer both checks.

1. A random sample of 80 of 1,200 school lunches contains 28 vegetarian choices. Identify the population, sample, variable, parameter, and statistic; compute the statistic.

2. Five students carry 3, 3, 4, 2, and 1 books. Their backpack weights are 6.2, 7.0, 6.8, 9.1, and 4.3 pounds. Their backpack colors are red, black, black, green, and gray. Classify each of the three variables as categorical or quantitative. For each quantitative variable, state whether it is discrete or continuous. Justify the distinction between book count and backpack weight.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 1.2, Examples 1.5, 1.6, and 1.8, printed p. 11-12. Adapted for this course; source data retained unless otherwise stated.

3. Forty visitors each choose one trail: 18 choose the river, 12 the forest, and 10 the ridge. Find each percentage and sector angle. Use the circle to draw a pie chart and label each sector with its trail and percentage.



S01.01-HW | Continued

4. A bar chart shows 46% and 50% with its vertical axis starting at 45%. Explain the problem and redesign the scale.

5. Write an investigative question about school travel that names a population, variable, and estimation goal. Identify data you would need.

6. A researcher reports "70% of the school favors the proposal" after 21 of 30 volunteers favor it. Correct the statement and explain which additional fact would matter for inference.

S01.02 | Two Categorical Variables and Association

CED 2.1, 2.2

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Hands-on option. Use this investigation or "Do the two bags have the same mix?," then continue to Independent application.

Equal counts, different stories

Use these illustrative survey counts: among 80 students who participate in a school sport, 48 prefer an early activity and 32 prefer a late activity. Among 120 nonparticipants, 48 prefer early and 72 prefer late. Do not ask students to reveal their own activities. Teams will produce different representations of this table.

Materials. Graph paper or spreadsheet; ruler.

Choose the denominator before dividing

Record counts first. Keep totals visible.

Group	Response 1	Response 2	Total
Group A			
Group B			
Total			

Conditional question: _____

Restricted denominator: _____ Relevant count: _____

Use the investigation's sport-participation and activity-preference counts. Label the groups and responses, then complete the counts and totals.

1. Complete the two-way table above. Label Group A as sport participants, Group B as nonparticipants, Response 1 as early, and Response 2 as late. Enter the four given counts and all row, column, and grand totals.

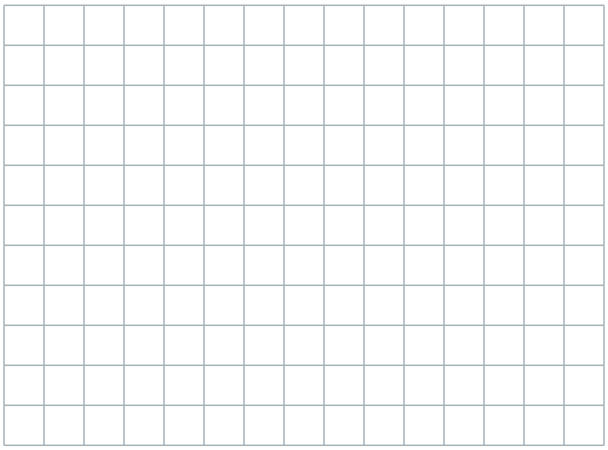
S01.02 | Classwork, continued

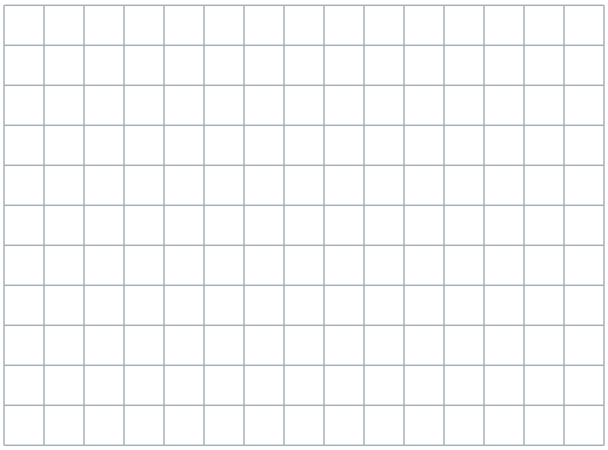
GUIDED APPLICATION, CONTINUED

2. Calculate the early-preference percentage within each participation group.

3. Among the students who prefer the early activity, calculate the percentage who participate in a sport. Write this as the conditional question below the table, then enter its denominator and numerator. Explain why its denominator differs from the denominator for early preference among participants.

4. Draw a segmented bar chart, then sketch a mosaic plot with correct group-width proportions.





5. Write a claim about association supported by two conditional percentages.

Discuss your results

- What does the area of a mosaic rectangle represent?
- Why do equal early counts not show equal preference rates?

S01.02 | Optional lab | Do the two bags have the same mix?

Optional guided application | 20 minutes

Compare the percentage of one color in each bag, using each bag's own total.

Materials. Two M&M's bags, or two bags of colored beads, labeled A and B. Provide a labeled tray or paper cup for each pair.

COLLECT YOUR DATA

1. Before opening the bags, choose a focal color and decide what denominator will make the comparison fair if the bags differ in size.

2. On the first row, circle your tray's bag letter and record your counts of focal and other pieces. Report both counts, then enter the separate class totals for bags A and B.

3. Individually calculate both within-bag percentages. Subtract B's percentage from A's and interpret the sign and size of the difference.

Source	Focal color	Other colors	Total pieces	Focal color (%)
Our tray (A / B)				
Whole bag A				
Whole bag B				

ANALYZE YOUR RESULTS

1. If you knew only the two focal-color counts, what information would you still need to compare the mix?

2. Write one sentence naming the color and both bags, reporting their percentages, and comparing them in percentage points.

Continue to this lesson's Independent application.

01

S01.02 | Practice and exit

INDEPENDENT APPLICATION

1. Of 100 employed students, 30 bike. Of 50 nonemployed students, 20 bike. Which group has the higher biking rate? Give both conditional percentages and their difference in percentage points.

2. Using the data in question 1, find the fraction of bikers who are employed and the joint fraction of all students who both bike and are employed.

3. A sample of cyclists contains 110 younger cyclists and 90 older cyclists. Of the younger cyclists, 38 prefer a hilly route; of the older cyclists, 52 prefer a hilly route. Compare the conditional percentages who prefer a hilly route. Does the sample show an association between age group and preference for a hilly route? Justify your answer.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 3.4, Try It 3.21, Table 3.7, printed p. 186. Adapted for this course; source data retained unless otherwise stated.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Group A has 24 successes among 40 trials; Group B has 30 among 75. Which has a higher observed success rate, and by how many percentage points?

S01.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 5. **Optional practice.** 6. **Prepare for the next meeting.** Read PR03 for S01.03 and answer both checks.

1. Illustrative survey: among 60 bus riders, 18 bring lunch and 42 buy lunch; among 90 other students, 45 bring and 45 buy. Construct a two-way table with all row totals, column totals, and the grand total. Find the overall proportion of students who bring lunch.

Draw your table here. Include headings and all rows needed for the data.

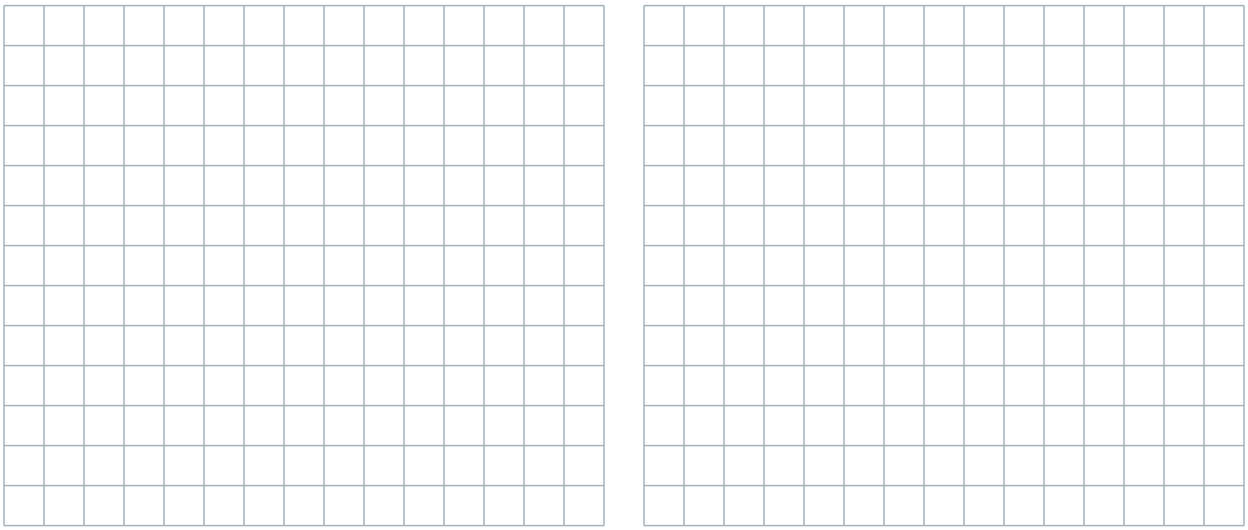
2. Using the lunch table, compare bring-lunch rates across travel groups and state whether there is an observed association.

3. Using the lunch table, find (a) the joint proportion who ride a bus and bring lunch and (b) the proportion of bring-lunch students who ride a bus. Explain the denominators.

01

S01.02-HW | Continued

4. Sketch a segmented bar chart for the lunch table. Describe a mosaic plot that also communicates the travel-group sizes.



5. A report says "Bus travel makes students buy lunch." Evaluate the claim using the lunch table and explain what was actually established.

6. In the lunch study in questions 1-4, 150 students were randomly sampled from the complete current school roster, and lunch choice and travel mode were recorded for each. Identify the observational unit and both variables. Define a population parameter the study could estimate and give its corresponding sample statistic.

S01.03 | Seeing a Quantitative Distribution

CED 1.5, 1.6

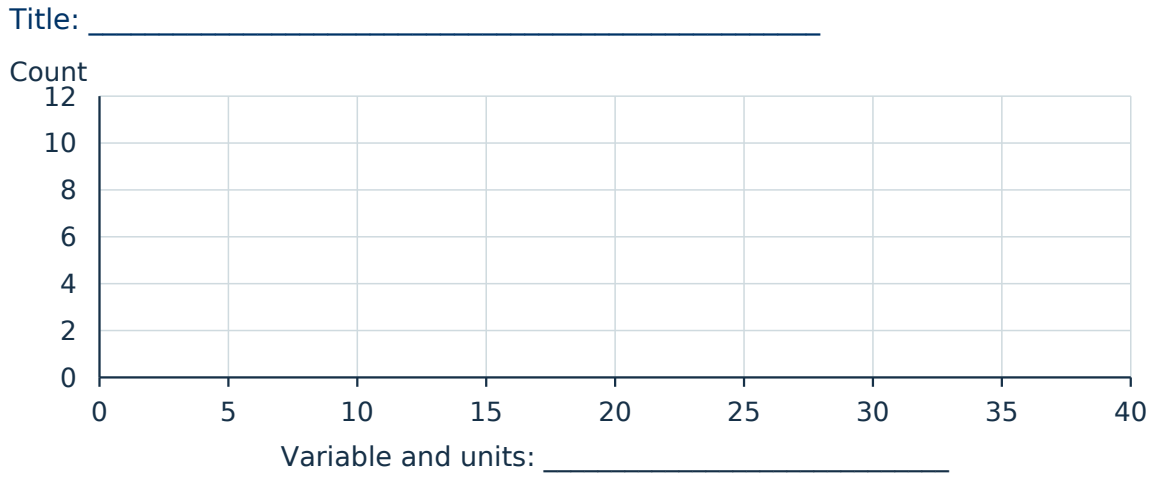
GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

One data set, three displays

Use these 12 illustrative wait times, in minutes: 6, 8, 9, 10, 10, 11, 12, 12, 13, 14, 16, 32. Groups will compare displays before agreeing on a written description.

Materials. Graph paper or spreadsheet.

Construct and label a histogram



Use these axes for the investigation's 10-minute bins. Label waiting time in minutes, and include the left endpoint of each bin.

1. Construct a dotplot and mark any interval with no observations between the main cluster and the largest value.

01

S01.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Use the supplied histogram axes to construct a histogram with bin boundaries 0, 10, 20, 30, and 40 minutes. Include the left endpoint and exclude the right endpoint of each bin. Label the horizontal axis Waiting time (minutes).

3. On the separate graph space below, construct a second histogram with boundaries 5, 10, 15, 20, 25, 30, and 35 minutes. Use the same left-inclusive, right-exclusive rule. Label both axes.

4. Write one feature visible in both histograms and one detail more visible in the dotplot.

S01.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

5. Write a contextual description with shape, approximate center, spread, and an unusual feature.

Discuss your results

- Does changing bins change the data?
- Can a histogram usually establish the exact median?

INDEPENDENT APPLICATION

1. The scores in 30 basketball games, in points, were 32, 32, 33, 34, 38, 40, 42, 42, 43, 44, 46, 47, 47, 48, 48, 48, 49, 50, 50, 51, 52, 52, 52, 53, 54, 56, 57, 57, 60, 61. Construct a stemplot with a key. Use the display to identify the interval of scores containing the greatest number of games.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 2.1, Try It 2.1, printed p. 67. Adapted for this course; source data retained unless otherwise stated.

01

S01.03 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. A distribution has most values near 90 and a few near 40. Which direction is the longer tail? Explain.

3. A histogram has 40 observations in [0, 10) and 10 in [10, 20). Can you find the exact mean? Justify your answer.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A histogram groups waiting times into five-minute intervals. Identify one detail that a dotplot of the original observations would show that the histogram might hide. Explain why the histogram does not preserve that detail.

S01.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 5, 6. **Optional practice.** 4. **Prepare for the next meeting.** Read PR04 for S01.04 and answer both checks.

1. Illustrative packing times are 4, 5, 5, 6, 6, 6, 7, 8, 9, 14 minutes. Make a dotplot and a stemplot with a key.

2. For those packing times, construct a histogram with bins [0, 5), [5, 10), [10, 15). Then use bins [4, 8), [8, 12), [12, 16). Explain what changes.

3. Describe the packing-time distribution using context, shape, center, spread, and an unusual feature.

01

S01.03-HW | Continued

4. A student says "The graph is skewed right because most bars are on the right." Correct the reasoning.

5. A survey records favorite trail as river, forest, or ridge. Explain why a histogram is inappropriate and name an appropriate display.

6. Two data sets have the same bin counts: four observations in [0, 10) and two in [10, 20). Give two possible data sets and explain one numerical feature that can differ.

S01.04 | Center, Variability, and Resistance

CED 1.7, 1.8

OPENING PREDICTION

Delays are 2, 4, 4, 6, and 9 minutes. If 9 becomes 24, predict how the mean, median, and spread will change. Give one reason.

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Move one observation

Use counters or written cards for 2, 4, 4, 6, 9. Treat these as invented delivery delays in minutes. Change only the largest value during the investigation.

Materials. Calculator or spreadsheet; five cards.

Track the calculation without losing units

Variable / units: _____ Mean: _____

Observed x	x − mean	(x − mean)²

Sum of squares: _____ n − 1: _____ Sample SD: _____

Use all five rows for the original delays 2, 4, 4, 6, and 9 minutes. Keep this table unchanged when comparing the revised data.

1. Compute mean and median of the original five times.

2. Use the five-row deviation table above for the original delays. Enter each value, its deviation from the mean, and its squared deviation. Verify that the deviations sum to zero, then calculate the sample standard deviation.

01

S01.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. Replace 9 with 24. Predict the direction of change in mean, median, range, and standard deviation before calculating.

4. Calculate the new mean, median, range, and sample standard deviation for 2, 4, 4, 6, and 24 minutes in the response space below. Retain the original values in the table so you can compare both sets of summaries.

5. Write an explanation of why the median behaves differently from the mean.

Discuss your results

- Why cannot ordinary signed deviations be averaged to measure spread?
- Does a large standard deviation imply a large mean?

INDEPENDENT APPLICATION

1. For the values 6, 12, 4, and 6, calculate the mean, median, range, and sample standard deviation. Show the mean calculation and identify the middle two ordered values.

2. A report gives s=8 dollars for the prices of a sample of meals. Interpret s.

3. Two routes have mean travel times 20 minutes; their standard deviations are 2 and 9 minutes. What practical difference do these summaries suggest? Justify your answer.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Why can a sample have mean 10 and standard deviation 0? Describe such a sample.

S01.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR05 for S01.05 and answer both checks.

1. Illustrative page-loading times are 3, 5, 7, and 13 seconds. Compute the sample mean, all four deviations, sample variance, and sample standard deviation.

2. Illustrative checkout waits are 2, 3, 3, 4, 8 minutes. Compute mean, median, and sample standard deviation. Choose summaries for a typical visit and explain.

3. A calculator displays Sx=4.2 and sigma-x=4.0 for observations from a sample. Which is the sample standard deviation? Explain the distinction.

01

S01.04-HW | Continued

4. In a sample of 60 houses, one is valued at $2,500,000, 29 are valued at $280,000, and the remaining 30 are valued at $315,000. Calculate the mean and median house value. Which measure better describes a typical house in this sample? Justify your answer.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 2.5, Try It 2.27, printed p. 100. Adapted for this course; source data retained unless otherwise stated.

5. For the categorical arrival counts car 22, bus 10, walk 6, bicycle 2, why is an average category code inappropriate? Give a useful numerical summary instead.

6. A sample of 20 trail walks has mean 42 minutes and s=6 minutes. Write a contextual interpretation of each, then explain why these numbers do not establish that most walks lie between 36 and 48 minutes.

S01.05 | Quartiles, Outliers, and Group Comparisons

CED 1.7, 1.8, 1.9

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

What can the box conceal?

Use two illustrative samples of eight delivery delays, in minutes: A=0, 0, 4, 4, 6, 6, 10, 10 and B=0, 1, 3, 4, 6, 7, 9, 10. Use the median-of-halves convention. The samples are constructed to investigate what a five-number summary can conceal.

Materials. Graph paper and calculator.

From quartiles to a modified boxplot

Variable / units: _____

Minimum	Q1	Median	Q3	Maximum

IQR = _____ Lower fence = _____ Upper fence = _____

Flagged observations: _____

_____

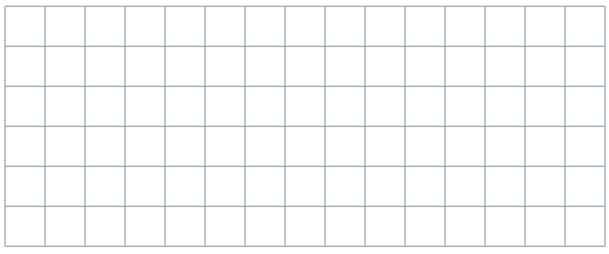
Label a scale, then mark observed nonoutlier endpoints and the quartiles.

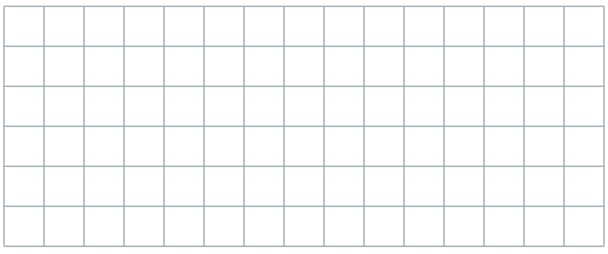
Keep the numerical fences separate from the observed whisker endpoints.

1. Use the organizer above for sample A. Record its five-number summary, IQR, fences, and any flagged observations. Calculate the same quantities for sample B in the response space below.

2. On the common numerical scale in the organizer, draw two labeled modified boxplots, one above the other, for samples A and B. Label the axis Delivery delay (minutes).

3. Draw a dotplot for each data set.





01

S01.05 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Identify a feature that differs in the dotplots but not the boxplots.

5. Write a statement explaining both the usefulness and limitation of the shared boxplot summary.

Discuss your results

- Does identical IQR guarantee identical standard deviation?
- Which graph would be preferable when the question concerns clusters?

INDEPENDENT APPLICATION

1. The following 11 annual salaries are in dollars: 33,000; 64,500; 28,000; 54,000; 72,000; 68,500; 69,000; 42,000; 54,000; 120,000; 40,500. Use the median-of-halves convention, excluding the overall median from both halves. Calculate the IQR and both fences. Identify any potential outliers and the whisker endpoints for a modified boxplot.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 2.3, Try It 2.13, printed p. 88. Adapted for this course; source data retained unless otherwise stated.

2. Route A has median 18 minutes, IQR 8; Route B has median 23 minutes, IQR 4. Compare the routes without claiming a cause.

3. A value is exactly $Q_3 + 1.5$ IQR. Is it flagged by the stated rule? Justify your answer.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A modified boxplot has upper fence 50 and largest nonoutlier observation 47. Where does the upper whisker end?

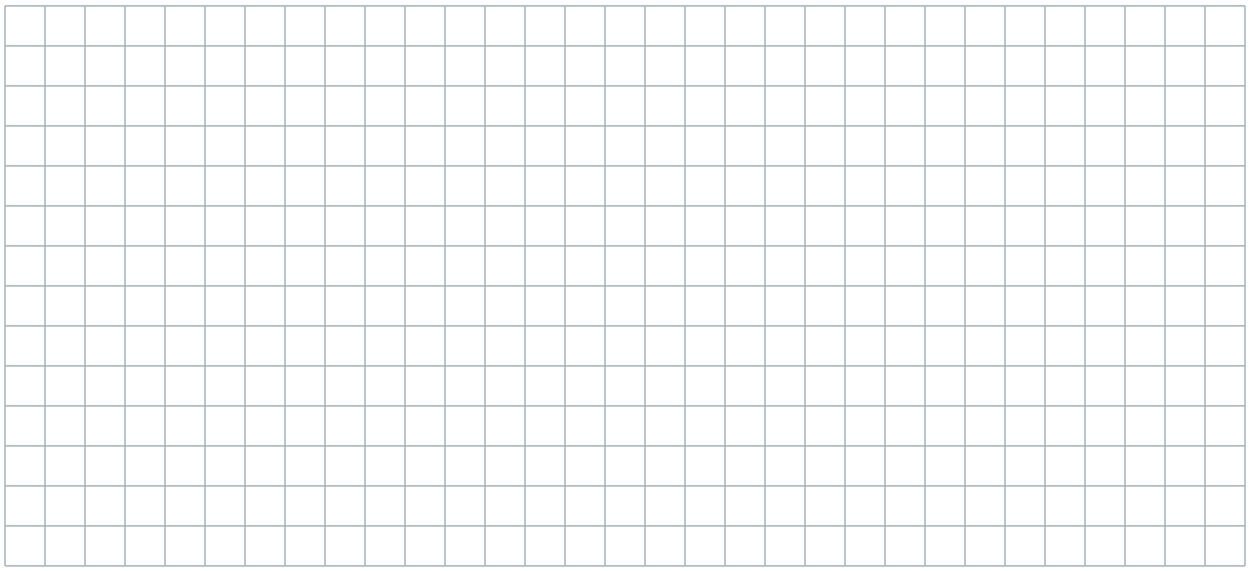
S01.05-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR06 for S01.06 and answer both checks.

1. Measured delays, in minutes, are 1, 2, 3, 4, 5, 6, 7, 8, 9, 20. Find the five-number summary, IQR, and 1.5 IQR fences.

2. Draw a modified boxplot for the delays in the previous question. State every plotted location and label units.



3. A sample has mean 50 and s=8. Apply the more-than-two-SD rule to values 33, 34, 65, and 67. Explain equality at a cutoff.

S01.05-HW | Continued

4. Use the back-to-back stemplot of North and South route times in the Student Reference, S01.05. Find the median and range of each sample. Compare the routes using both values. Name a detail the ranges conceal.

5. For a sample with mean 18 minutes and s=3 minutes, a classmate calls 24 minutes an outlier because it is exactly two SD above the mean. Evaluate the claim under the stated strict rule and discuss whether another rule might differ.

6. A sample comparison finds a larger median sleep duration for one grade. Write one appropriate descriptive claim and explain why the graph alone does not show that grade level causes longer sleep.

S01.06 | Relative Standing and Changing Units

CED 1.7, 1.9

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Change the ruler, keep the pattern

Use invented measurements 2, 4, 6, 8, 15 centimeters. Let y=3+2 x and w=20-x.

Materials. Calculator or spreadsheet; graph paper.

Record a transformation and its consequences

Transformation: y = _____

Quantity	Original	Transformed
Mean		
Standard deviation		
Selected observation		
Standardized score		

Predict first: what changes in location, spread, order, and shape?

Complete the prediction before calculating the transformed summaries.

1. Find the mean and sample SD of x.

2. Use the organizer above for y=3+2x. Predict the effect on center, spread, order, and shape. Then transform all five values, calculate the new mean and sample standard deviation, and check your predictions against the transformation formulas.

3. For w=20−x, use the response space below. List and sort the five transformed values. Calculate their mean and sample standard deviation, then describe how the order and shape compare with the original data.

01

S01.06 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Compute the percentile rank of x=6 using the at-or-below convention.

5. Compute the sample-based z-score of x=8 and its corresponding y and w values.

Discuss your results

- Why does adding 3 not affect SD?
- Does standardizing guarantee a normal distribution?

INDEPENDENT APPLICATION

1. For finishing times in a 100-meter dash, the third quartile is 11.5 seconds. Interpret this quartile in context. Explain why a higher percentile of finishing time does not necessarily represent a better performance.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 2.3, Try It 2.19, printed p. 93. Adapted for this course; source data retained unless otherwise stated.

2. A race time is 52 seconds in a population with mean 60 and SD 4. Find z and explain whether negative means poor performance.

3. Prices have mean 40 dollars and SD 6 dollars. A store applies y=0.8 x+2 to every price. Find new mean and SD.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A data set is standardized. A student says its shape must now be normal. Is that correct? Explain.

S01.06-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 5, 6. **Prepare for the next meeting.** Read PR07 for S02.01 and answer both checks.

1. Ordered sample values are 2, 3, 3, 5, 7, 8, 10, 12. Find empirical percentile ranks of 3 and 8 using the stated convention; explain the role of ties.

2. Population A has mean 100 and SD 15; B has mean 40 and SD 4. Compare values 124 in A and 47 in B by z-score.

3. Temperature data have mean 18 C, median 17 C, and SD 3 C. Convert the three summaries to Fahrenheit using F=32+1.8 C.

01

S01.06-HW | Continued

4. A data set has mean 12, SD 5, $Q_1=8$, $Q_3=16$. For $y=30-2x$, find the new mean, SD, Q_1, and Q_3.

5. A student says $z=1.5$ means that 75% of observations are below the value. Explain why a z-score alone does not determine a percentile. Give the correct interpretation of $z=1.5$.

6. For each observed vehicle trip to campus, a survey records vehicle color and travel time in minutes. Which variable can be standardized meaningfully? Explain your answer and identify the observational unit.

Unit 1 | Cumulative synthesis

1. An illustrative survey of 200 students records grade, arrival mode, and travel time. Identify a categorical and a quantitative variable and write an investigative question with a defined population.

2. Among 80 students in group A, 48 prefer option 1; among 120 in group B, 48 prefer option 1. Compare groups and compute the joint proportion in A and preferring 1.

3. Use data 2, 4, 4, 6, 9. Find mean, median, and sample SD; interpret the SD if the data are minutes.

4. Use data 6, 7, 8, 8, 9, 10, 11, 12, 13, 30. Find IQR, fences, and whisker endpoints under the stated convention.

5. Population summaries are mean 50 and SD 10. Find z for 65. Then transform all values by y=100-2 x and determine transformed mean, SD, and z for the transformed 65.

6. A report claims that the school's commuting habits cause a difference in test scores because survey groups have different means. Identify two distinct limits.

Unit 02 | Collecting Data

Good analysis cannot repair every weakness in data collection. This unit connects the investigative question to the population, sampling plan, measurements, and conclusions. You will describe reproducible random selection and random assignment, distinguish bias from ordinary sampling variation, and choose between completely randomized, block, and matched-pairs experiments. The central distinction is practical: random sampling supports population generalization, while random assignment supports causal interpretation when the experiment is well designed. Both require careful implementation. Activities use invented populations or low-risk classroom tasks, with voluntary participation and an equivalent alternative when actual data are collected.

CED topics: 1.10, 1.11, 1.12, 1.13

IN THIS UNIT

Lesson	Focus
S02.01	Choosing a Random Sample
S02.02	Bias, Observational Studies, and Ethical Questions
S02.03	Designing a Completely Randomized Experiment
S02.04	Blocking, Matched Pairs, and Scope of Inference

Choosing orchard sampling units

Possible sampling frame	Who or what can be sampled?
Roster of trees	A tree is selected first. A fruit selection rule is still needed.
Fruit within reach beside the path	Only fruit in that accessible group can be selected.

Decide whether the target is trees or individual pears.

LOCAL QUESTION

How could an orchard sample avoid favoring fruit that is easiest to reach? Compare random selection across trees with convenience selection beside the path.

S02.01 | Choosing a Random Sample

CED 1.10, 1.11

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Sample a paper population

Invented population: 40 numbered cards. Cards 1 through 20 have value 0; cards 21 through 40 have value 1. The true population proportion of 1 s is 0.50. Compare a convenience sample with repeated random samples of 8 cards.

Materials. Forty cards or slips; random-number generator or shuffled deck; recording sheet.

Write a sampling plan someone else could implement

Make the selection procedure reproducible.

Planning decision	Your specification
Target population	
Sampling frame	
Selection mechanism	
Why this method?	

Check: labels, sample size, duplicates, and stopping rule.

For a survey of people, also plan for nonresponse.

Describe how to select eight distinct cards from the 40-card population. Give the complete mechanism, not just the method's name.

1. Take the first 8 numbered cards and compute the proportion of 1 s. Explain the selection problem.

2. Use the sampling-plan organizer to describe a simple random sample of 8 distinct cards from the 40-card population. State the labels, random selection method, duplicate rule, and stopping rule. Carry out the plan and record the proportion of selected cards with value 1. Return all eight cards before selecting another sample.

S02.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. Repeat the entire random sampling procedure five times or pool class results. Record one proportion per sample.

4. Compare a stratified sample that chooses 4 random cards from 1 through 20 and 4 from 21 through 40.

5. Explain what this artificial example demonstrates and why real strata rarely eliminate all variation.

Discuss your results

- Must every random sample give exactly 0.50?
- What should change if the data values within each stratum vary?

INDEPENDENT APPLICATION

1. A district randomly selects 3 of 20 schools and surveys every teacher in those schools. Name the design and explain a limitation.

2. A roster has 500 students. Describe a systematic sample of 25 and identify one risk.

3. A population is 80% seniors and 20% juniors. An investigator samples 20 from each group. Is the unweighted overall sample percentage automatically appropriate? Justify your answer.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Every student has an equal chance of inclusion in a sample consisting of one randomly selected homeroom. Is the sample necessarily an SRS of students?

S02.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR08 for S02.02 and answer both checks.

1. Describe an SRS of 30 from a complete 900-student roster. Include labeling, random selection, duplicates, and the connection back to students.

2. A school has 200 ninth graders, 150 tenth graders, 100 eleventh graders, and 50 twelfth graders. Allocate a proportional stratified sample of 40 and describe selection.

3. A counselor uses a complete high-school roster, assigns a unique integer to each student, and uses a random-number generator to choose 50 distinct integers, with every set of 50 labels equally likely. Another student interviews classmates in their algebra class to estimate the mean number of pairs of jeans owned by all students at the school. Identify each sampling method and explain why the second may produce a biased estimate.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 1.2, Example 1.12, parts e-f, printed p. 22-23. Adapted for this course; source data retained unless otherwise stated.

S02.01-HW | Continued

4. A sample of 15 is drawn systematically from 150 items. If the random start is 7, list the first four and final selected positions.

5. A student says "Our random sample differs from the population, so random sampling is biased." Correct the reasoning.

6. An SRS of 50 students records 18 bike commuters. Identify the statistic and a parameter it estimates, then explain why adding 500 volunteers is not a guaranteed improvement.

S02.02 | Bias, Observational Studies, and Ethical Questions

CED 1.10, 1.12

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Diagnose the process, not the percentage

Consider four proposed surveys intended to estimate support among all current SMS students for extending library hours. A surveys only students entering the gym; B randomly selects 100 students from the complete roster but 30 do not answer; C asks, "Do you support our excellent new plan?"; D posts an open invitation to vote online. These are classroom scenarios, not reports of actual school surveys.

Materials. Four scenario cards or a projected list.

Audit a data-collection plan

Investigative question: _____

Process step	Potential problem	Mechanism / repair
Coverage		
Participation		
Measurement		
Privacy / consent		

If you predict a direction of bias, state the contextual reason.

Use the organizer to explain a possible bias rather than merely name it.

1. For each scenario A-D, identify a potential source of bias and explain how the selection or response process could systematically distort the estimated support. Use the Coverage row of the organizer for A, Participation for B and D, and Measurement for C. Use the Privacy/consent row to state one safeguard common to all four plans.

2. For each scenario, say whether the direction of bias can be determined from the stated facts.

3. Rewrite C as a neutral question with a clearly defined response.

S02.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Propose a practical improvement for each design.

5. Write one valid estimation question and one testing question about a nonsensitive school issue.

Discuss your results

- Could a census still suffer response bias?
- Why should the original investigative question be retained after an unexpected result?

INDEPENDENT APPLICATION

1. A researcher wants to investigate whether birth order affects personality. Explain why birth order cannot be assigned in a randomized experiment. Describe one difficulty in drawing a cause-and-effect conclusion from an observational comparison of first-born and later-born children.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 1.4, Example 1.21, printed p. 36. Adapted for this course; source data retained unless otherwise stated.

2. A selected survey participant leaves every question unanswered. Another understates screen time because the interviewer is a teacher. Distinguish the problems.

3. A survey samples from a complete roster but only 70% respond. Does the complete frame eliminate all selection concerns? Justify your answer.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A study finds students who choose tutoring have higher exam scores. Give a plausible confounder and explain both required connections.

S02.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR09 for S02.03 and answer both checks.

1. A city surveys drivers at a downtown parking garage to estimate the proportion of all residents who bike. Identify a potential bias and explain a plausible direction.

2. Of 200 randomly selected residents, 120 reply and 84 support a proposal. Compute the respondent support rate and response rate. Explain why they answer different questions.

3. Rewrite "Don't you agree our unsafe crossing urgently needs expensive improvements?" as a neutral measurable question. Explain two problems with the original.

S02.02-HW | Continued

4. A study follows 40 volunteer runners for six months while recording their own chosen training and race times. Classify prospective/retrospective and observational/experimental; state a limit.

5. Write an estimation question about the mean number of minutes students spend traveling to school. Then write a directional testing question about the same parameter.

6. A researcher increases a voluntary website poll from 1,000 to 10,000 responses and claims bias is now impossible. Explain why this reasoning fails and suggest a better improvement.

S02.03 | Designing a Completely Randomized Experiment

CED 1.10, 1.13

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Make the allocation reproducible

Use 24 invented participant labels. Plan a comparison of two neutral puzzle layouts; no real student performance data are required. A second design has two layouts and two font sizes.

Materials. Twenty-four numbered slips or a random-number generator.

Specify the experiment before collecting outcomes

Experimental units / recruitment	**Treatments / factor levels**
Random assignment mechanism	**Response / measurement rule**
Direct controls / blinding	**Independent units per treatment**

The organizer separates the study population, treatment mechanism, measurement, and design safeguards.

1. Describe and carry out a random allocation of 12 labels to layout A and 12 to B. Record the resulting groups.

2. For the two-layout experiment, complete the corresponding rows of the organizer. Identify the experimental units, the factor and its two levels, and a response variable with a precise measurement rule. Record the random allocation from question 1 in the assignment row.

S02.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. Name three conditions to hold constant and explain how each could otherwise affect the response.

4. For the two-layout, two-font experiment, list all treatments and allocate six units to each by one complete randomization.

5. Critique assigning the first 12 arrivals to A and the next 12 to B.

Discuss your results

- What does replication mean in this design?
- Which participants or personnel could realistically be blinded?

INDEPENDENT APPLICATION

1. A plant experiment has two fertilizers and three watering schedules. How many treatments are there, and what are they? Justify your answer.

2. Twenty similar pots, each containing one plant, are randomly assigned to two fertilizers, with 10 pots receiving each fertilizer. Each plant is measured weekly for six weeks. Identify the experimental unit and the number of independently assigned units per treatment. Explain why there are not 60 independent experimental units per treatment.

3. An experiment assigns a new reading method to volunteers at random. State what random assignment helps establish and what it does not guarantee.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A researcher gives the new method to the strongest students and the old method to the others. Explain the main problem.

S02.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR10 for S02.04 and answer both checks.

1. Design a randomized experiment with 36 volunteers comparing three neutral study-card formats. State units, treatments, response, assignment, replication, and direct controls.

2. An experiment uses two light levels and three watering amounts on plants. Name the number of factors, levels per factor, and treatments.

3. A team measures one machine 100 times under method A and one other machine 100 times under B. Explain why this is weak replication for comparing methods.

S02.03-HW | Continued

4. Researchers want to compare drivers' reaction times with and without a texting task. Identify the explanatory variable and a measurable response variable. Explain why assigning participants to text while driving on public roads would be inappropriate. Describe how a simulator could preserve the comparison while reducing risk, and identify who could be blinded.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 1.4, Try It 1.21, printed p. 36. Adapted for this course; source data retained unless otherwise stated.

5. A random assignment produces unequal baseline averages. Does that prove the randomization failed? Explain, and state why direct control still helps.

6. Return to bias: an experiment recruits only enthusiastic volunteers through a website. It then assigns treatments randomly. Which concern remains for generalization, and which design feature helps causal interpretation?

S02.04 | Blocking, Matched Pairs, and Scope of Inference

CED 1.10, 1.13

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Which design matches the question?

Compare two neutral page layouts using invented participants. Scenario A has 24 people of mixed reading experience. Scenario B lets each person use both layouts with two equivalent texts. Illustrative paired time differences, new minus standard, are −7, −2, −1, 0, 1, and −3 minutes.

Materials. Scenario cards; calculator.

Preserve the pairs in the data record

Subtraction order: _____

Pair / unit code	Observed difference	Random sign	Simulated difference

Observed mean: _____ Simulated mean: _____ Units: _____

One row per pair. Preserve pairs in every randomization repetition.

Use the six illustrative paired differences from the investigation. Record one no-effect randomization while preserving the pairs.

1. Propose a randomized block design for A using reading experience. State where randomization occurs.

2. Propose a matched-pairs design for B. Explain how to handle order and text differences.

S02.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. Enter the six supplied new-minus-standard differences in the Observed difference column. Calculate their mean, record it beneath the table, and interpret its sign in terms of page-layout completion time.

4. Carry out one no-layout-effect randomization in the table. For each of the six pairs independently, use a fair coin to choose +1 or −1. Enter that multiplier in Random sign, multiply the observed difference by it, and enter the result in Simulated difference. Calculate the simulated mean. Explain why the pairs remain intact.

S02.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

5. State what conclusions would change if participants were randomly sampled from a defined population rather than volunteers.

Discuss your results

- Why is shuffling all 12 times freely inappropriate for the paired design?
- Can a small tail proportion repair poor measurement or a biased sample?

INDEPENDENT APPLICATION

1. A study matches 20 pairs of similar trees, then flips a coin in each pair to assign fertilizer A to one tree and B to the other. Describe the design and benefit.

S02.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. A random sample of residents reports exercise habits and health. Which conclusion is supported more directly: population association or causal effect? Justify your answer.

3. In 2,000 illustrative no-effect randomizations, 42 differences are at least as large as a prespecified positive observed difference. Estimate the tail probability and interpret without saying the null is probably false.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A blocked experiment assigns every experienced participant to A and every inexperienced participant to B. What is wrong?

S02.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 5, 6. **Prepare for the next meeting.** Read PR11 for P01 and answer both checks.

1. Design a randomized block experiment comparing two fertilizers on 40 plants, 20 small and 20 large. Explain why initial size could be a useful block.

2. Using the maze-study design from the reference, suppose 16 volunteers will each try both a floral-scented and an unscented mask on equivalent sets of mazes. Describe a complete random assignment of eight volunteers to each treatment order. Explain why the analysis should use within-person differences rather than treat the two sets of times as independent.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 1.4, Example 1.20, design application, printed p. 35-36. Adapted for this course; source data retained unless otherwise stated.

3. Five paired time differences, new minus standard, are −2, −1, 0, −8, and 1 minutes. Compute the mean difference and interpret it if smaller time is desirable.

S02.04-HW | Continued

4. For each study, state whether population generalization and causal interpretation are supported by the design: random sample with no assigned treatment; volunteers with random treatment assignment; random sample with random treatment assignment.

5. An observed revised-minus-standard mean is -2.4 minutes. In 1,500 illustrative within-block randomizations, 60 differences are -2.4 or smaller. Estimate the relevant tail proportion and describe one simulation repetition.

6. A student says "The randomization p-value is 0.04, so there is a 96% chance the treatment works for everyone." Identify two errors.

Unit 2 | Cumulative synthesis

1. Describe an SRS of 20 from a 300-person roster and explain why taking one randomly selected 20-person club is generally not an SRS of people.

2. Distinguish undercoverage, nonresponse, and response bias using a survey about library use.

3. Explain why self-selected tutoring and exam score can be confounded by motivation.

4. Design a randomized block experiment with 30 novice and 30 experienced users comparing two interfaces.

5. An experiment uses volunteers but random treatment assignment. State a defensible conclusion if convincing evidence favors the treatment.

6. In a paired experiment, what must a no-effect randomization preserve?

P01 | How much can a sample change the story?

Course project • Meeting 11. How well does a random sample of ten books estimate the mean page count of this forty-book collection?



A collection worth investigating

Work in groups of three. Use this illustrative population of page counts, labeled 01-40 in the order shown: 80, 96, 104, 112, 120, 128, 144, 152, 160, 168, 176, 184, 192, 200, 208, 216, 224, 232, 240, 248, 256, 264, 272, 280, 288, 304, 320, 336, 352, 368, 384, 400, 432, 464, 496, 528, 576, 640, 720, 880. These are fictional book data. They form the entire population for this investigation. Begin without computing the full-population mean.

1. Before sampling, write the population, variable, parameter, and a prediction about the shape. Explain why the question calls for data.

2. Write a reproducible SRS protocol: label 01-40, select ten distinct labels with a random-number generator, ignore repeats, and retain the books at the selected labels. Save the labels in draw order before inspecting their page counts. Record the generator and, if it accepts one, its seed.

3. Collect one SRS of size ten. Make a labeled dotplot and compute its mean, median, and sample standard deviation. Describe the distribution in context. Do not delete an extreme observation solely because it is unusual.

P01 | Analysis and reporting

4. Combine the class sample means in one display. What varies across groups even though everyone used the same method? Now compute the population mean from all forty values and compare your estimate with it.

5. Construct a deliberately biased method, such as choosing the ten shortest books. Calculate its mean. Explain why increasing the number of samples from that method would not cure the method's bias.

6. Write a one-page evidence brief that distinguishes variability from bias. Include your sample labels, graph, calculations, the class comparison, and one limitation. End with an individual 100-word explanation of why a random sample need not look perfectly representative.

Required product

One group evidence brief with the saved sampling record, plus an individual explanation. Share only within class; no external publication is needed.

Assessment criteria

- 4 points: investigative question, population/parameter, and reproducible random selection.
- 4 points: accurate display, mean/median/SD, labels and page units.
- 4 points: evidence-based comparison of the sample estimates with the population and biased method.
- 4 points: clear distinction between random variability and systematic bias, including a justified limitation.
- 4 points: individual explanation demonstrates reasoning independently of the group.

Unit 03 | Probability

Probability connects a chance model to the patterns expected from repeated observations. This unit begins with simulation so students must define a trial, an event, and a defensible model before using formulas. It then develops complements, overlapping events, conditional probabilities, independence, and multi-stage trees. Calculations are paired with interpretations and checks on the model. The same habits return in inference: specify the assumption, determine how unusual a result would be, and limit the conclusion to what the study supports. All supplied classroom datasets and model rates are illustrative rather than factual claims about the school or region.

CED topics: 2.3, 2.4, 2.5, 2.6, 2.7

IN THIS UNIT	
Lesson	Focus
S03.01	Chance models and reproducible simulation
S03.02	Events, complements, and overlapping categories
S03.03	Conditional probability and independence
S03.04	Multi-stage chance processes and probability trees

A model for helmet use

One rider	One group of five riders
Record H or N for an eligible rider	Count how many of the five outcomes are H.

For a fixed-chance independent model, each new rider has the same chance of wearing a helmet regardless of previous riders.

> **LOCAL QUESTION**
>
> If a model gives each passing rider a fixed chance of wearing a helmet, what assumptions would a five-rider simulation require?

S03.01 | Chance models and reproducible simulation

CED 2.3

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

What counts as one trial?

Suppose each free throw independently has probability 0.60 of a make. One trial consists of five attempts. Let X be the number of makes in that trial. A qualifying trial has at least four makes. Generate your own results unless your teacher supplies the clearly labeled demonstration total in question 3.

Materials. A calculator random-integer generator or ten slips labeled 0-9 in a cup; replace slips after every draw.

Simulation recording organizer

Record one row per complete five-attempt trial.

Trial	Five random digits	Makes, X	Qualifies?
1			
2			
3			
4			
5			

Event rule: _____ Estimate = _____ qualifying / _____ total

Use this organizer after defining the digit assignment and the event. Add more rows or pool compatible class trials.

1. Before generating numbers, predict whether at least four makes in five attempts is common or rare. State the event.

2. Use digits 0-5 for a make and 6-9 for a miss. Each partner generates five complete trials on their own five-row organizer. In each row, record five independently generated digits, the number of makes X, and Yes or No in Qualifies? according to whether X is at least 4. Combine the two partners' results for ten trials.

3. Pool the class trials after verifying that every group used the same model. Calculate the number of qualifying five-attempt trials divided by the total number of five-attempt trials. If live results are unavailable, use this demonstration result instead: 68 qualifying trials out of 200. State which data you used.

S03.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Explain how using digits 0-6 for success would change the model. Explain why keeping the same five digits for every repetition would fail.

Discuss your results

• Does doubling the number of simulated trials guarantee an estimate closer to the true probability?

• What evidence would be needed to decide whether a real shooter has success probability 0.60?

INDEPENDENT APPLICATION

1. A student reports 600 makes among 1,000 simulated shots. Does 0.60 estimate the chance of at least four makes in five? Explain.

2. Design a digit simulation for the chance that at least one of three independent orders is late when each order has late probability 0.20.

3. A box contains one white ball and one red ball. A ball is selected at random, replaced, and the box is mixed before a second selection. Describe a random-digit simulation to estimate the probability that the two selected balls have different colors. State what one trial represents, what is recorded, and how the estimate is calculated.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 3.2, Try It 3.7, part b, with simulation extension, printed p. 175. Adapted for this course; source data retained unless otherwise stated.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A simulation produces 12 qualifying trials out of 400. Interpret the result and identify one limitation.

S03.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR14 for S03.02 and answer both checks.

1. A shipment has independent damage probability 0.08 per parcel. Design a two-digit simulation for the chance of at least one damaged parcel among five, including the stopping rule and recorded statistic.

2. In 500 five-parcel simulations, 166 contain at least one damaged parcel. Calculate and interpret the estimate. Is it a probability for one parcel?

3. A random integer generator returns all integers 1-12 equally often in the long run. Describe an exact assignment for a probability of 0.25 and explain why assigning 1-4 is wrong.

S03.01-HW | Continued

4. A student says, "Our first 100 trials gave 0.31, so 1,000 trials must give an estimate closer to the true value." Correct the claim.

5. Recall: A school uses only students who volunteer for a lunchtime survey. Explain why simulating that survey 10,000 times cannot make it representative of all students.

6. A simulation models a claimed process and produces 2,000 counts. Seven simulated counts are at least as large as the actual observed count. Calculate the estimated probability of a result at least this large under the model. Explain what this suggests about the claim, assuming the model assumptions and simulation are appropriate.

S03.02 | Events, complements, and overlapping categories

CED 2.4, 2.5, 2.7

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Hands-on option. Use this investigation or "Doubles or a sum of eight," then continue to Independent application.

Build a complete event table

Use an illustrative set of 40 numbered student cards. Cards 1-18 are labeled bike; cards 11-24 are labeled bus. All remaining cards carry neither label.

Materials. Paper cards or a list of integers 1-40.

Four-cell event organizer

Investigation: cards 1–40

Bike labels: 1–18. Bus labels: 11–24.
Sort every card into exactly one cell before calculating probabilities.

	Bus	No bus	Total
Bike			
No bike			
Total			

The card-label assignments are given information. Cell counts and totals are left blank for the investigation.

1. Use the four interior cells of the organizer to record the counts for bike and bus, bike and no bus, no bike and bus, and neither. Each card must enter exactly one cell. Complete all margins and check that the grand total is 40.

S03.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Calculate the probabilities of bike, bus, both, and at least one when one card is selected uniformly.

3. Compare the sum of the two individual probabilities with the probability of their union. Identify the extra counted cards.

4. Now change the bus labels to cards 19-32. Keep the original table unchanged and calculate the new intersection and union in the response space below. State whether the revised events are mutually exclusive and justify your answer.

Discuss your results

- Why must the four cell counts sum to 40?
- When may two event probabilities be added without subtracting an overlap?

S03.02 | Optional lab | Doubles or a sum of eight

Optional guided application | 20 minutes

Compare the observed probability of an inclusive union with its value under an independent fair-dice model.

Materials. Two distinguishable six-sided dice per pair, one roll tally per pair, and a board for class totals.

COLLECT YOUR DATA

1. Before rolling, identify an ordered pair that is both doubles and a sum of eight. Decide which single cell will receive its tally.

2. Roll both dice 12 times per pair. For each roll, enter one tally in the two-by-two table, then add the margins and check that the grand total is 12.

3. Pool the pairs' cell counts. Individually find the fraction of class rolls that were doubles, had a sum of eight, or both. Compare this fraction with the fair-dice model.

Rolled pair	Sum is 8	Sum is not 8	Total
Doubles			
Not doubles			
Total			

ANALYZE YOUR RESULTS

1. Out of 36 equally likely ordered outcomes, how many are doubles, how many sum to eight, and how many meet both conditions?

2. In one sentence, report the class fraction for doubles or a sum of eight, compare it with the model probability, and explain why the overlap is counted once.

Continue to this lesson's Independent application.

S03.02 | Practice and exit

INDEPENDENT APPLICATION

1. Roll one fair six-sided die. Let A be the event that the result is prime and B the event that the result is odd. List the outcomes in each event, their intersection, and their union. Calculate P(A and B) and P(A or B), and state whether the events are mutually exclusive.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 3.5, Try It 3.28, printed p. 194. Adapted for this course; source data retained unless otherwise stated.

2. Of 100 orders, 25 are late, 18 are damaged, and 7 are both. Find the chance a randomly selected order is neither.

3. A student gives probability 1/11 for a sum of 7 on two fair dice. Explain the error and calculate the correct value.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

P(A)=0.35, P(B)=0.50, P(A or B)=0.65. Are A and B mutually exclusive? Justify.

S03.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR15 for S03.03 and answer both checks.

1. A club lists 60 students: 28 like hiking, 25 like cycling, and 10 like both. Find counts and probabilities for hiking only, cycling only, both, and neither.

2. Using the club data, calculate the probability of at least one preference and exactly one preference. Explain why they differ.

3. For a randomly selected bookstore customer, the probability of buying a novel is 0.60, the probability of buying nonfiction is 0.40, and the probability of buying both is 0.20. Calculate the probability of buying at least one of these two types of books and the probability of buying neither. Show how you account for the overlap.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 3.5, Try It 3.30, parts a-b, with complement extension, printed p. 196. Adapted for this course; source data retained unless otherwise stated.

S03.02-HW | Continued

4. For each event, state its complement in words: (a) at least two defective items; (b) no missed assignments; (c) more than five arrivals.

5. Recall: Design a digit model for three independent events each with probability .30. State what to record to estimate at least one occurrence.

6. Someone claims P(A)=.70, P(B)=.60, and P(A and B)=.10. Explain why the claims cannot all hold.

S03.03 | Conditional probability and independence

CED 2.6, 2.7

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Change the denominator

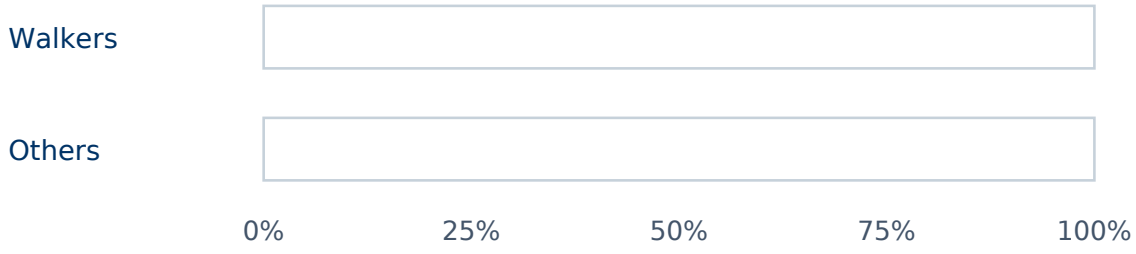
An illustrative survey records travel group and support for a proposed schedule change. Among walkers, 54 support the change and 36 oppose it. Among other commuters, 48 support it and 62 oppose it. Organize these four counts into a two-way table; each person appears in exactly one cell.

Materials. A two-way table with the four counts 54, 36, 48, 62.

Build the conditional percentage display

Given counts	Support	Oppose	Total
Walkers	54	36	
Others	48	62	
Total			

Shade each bar by its own conditional percentages.



Only the supplied respondent counts are shown. Complete the row totals, column totals, and grand total; then calculate percentages, and label the two bars.

1. Complete row, column, and grand totals.

S03.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Suppose one of the 200 respondents is selected at random. Calculate (a) the probability the person walks and supports the change, (b) the probability the person supports the change given that the person walks, and (c) the probability the person walks given that the person supports the change. Identify the denominator in each calculation and interpret the three answers in context.

3. Compare support percentages for walkers and other commuters. Draw two equal-length 100% bars using those percentages.

S03.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Keep the row totals 90 and 110, but change the support total to 100. Construct a new integer table in which walking and support are independent.

Draw your table here. Include headings and all rows needed for the data.

Discuss your results

- Why are equal-length percentage bars better than raw counts for this comparison?
- Could a survey using volunteers establish the same pattern for all school students?

INDEPENDENT APPLICATION

1. In a group of 80, 30 play an instrument and 18 of those 30 also play a sport. Find the probability of a sport given an instrument.

S03.03 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. At a library, let B be the event that a randomly selected visitor checks out a book and D the event that the visitor checks out a DVD. Suppose P(B)=0.40, P(D)=0.30, and P(B and D)=0.20. Calculate P(B given D) and P(D given B). Determine whether the events are independent and whether they are mutually exclusive. Justify both decisions.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 3.2, Try It 3.10, printed p. 177. Adapted for this course; source data retained unless otherwise stated.

3. Two positive-probability events are mutually exclusive. Can they be independent? Explain without relying on a slogan.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Among 60 runners, 18 volunteer; among 40 nonrunners, 12 volunteer. Are running and volunteering independent in this 100-person group?

S03.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 5, 6. **Optional practice.** 4. **Prepare for the next meeting.** Read PR16 for S03.04 and answer both checks.

1. Of 40 morning visitors, 28 purchase and 12 do not; of 60 afternoon visitors, 30 purchase and 30 do not. Construct a two-way table with all totals. Find P(purchase), P(purchase given morning), and P(morning given purchase).

Draw your table here. Include headings and all rows needed for the data.

2. Use the visitor table to decide whether time group and purchase are independent within these 100 visits. Give numerical evidence and one limit.

3. For events A and B, P(A)=.55, P(B)=.40, and P(A given B)=.25. Find the intersection and union.

S03.03-HW | Continued

4. A student compares 60 successes in a group of 100 with 45 successes in a group of 50 and concludes the first group performs better. Correct the comparison.

5. Recall: P(A)=.30 and P(B)=.50 for disjoint events. Find P(neither) and P(A given B).

6. Construct a two-way table with row totals 40 and 60 in which the success probability is .25 in both rows. Verify independence.

Draw your table here. Include headings and all rows needed for the data.

S03.04 | Multi-stage chance processes and probability trees

CED 2.6, 2.7

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

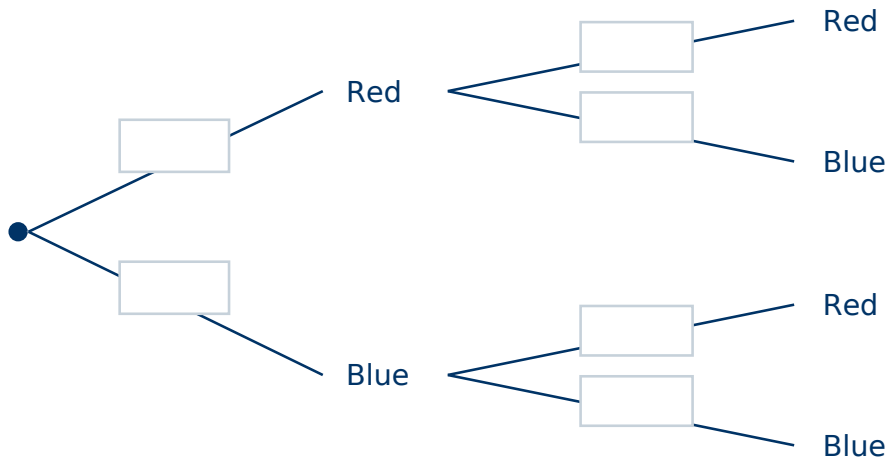
Two draws, two models

Use a bag containing 4 red and 6 blue counters. Study the event exactly one red in two draws.

Materials. Ten slips or counters, paper for a two-level tree, random selection method.

Label the probability tree

Given: 4 red, 6 blue; two draws without replacement.



Fill branch probabilities. Circle the complete paths for the event you are studying.

The outcomes and original bag composition are given. Complete every branch probability before combining paths.

1. Complete the supplied probability tree for two random draws without replacement from the bag of 4 red and 6 blue counters. Label both first-stage probabilities and all four second-stage probabilities. At each second-stage branch, use the contents remaining after the first draw.

S03.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Calculate the probability of exactly one red by adding the red-blue and blue-red path probabilities.

3. For draws with replacement, sketch a new tree or write both path products in the response space below. Replace the first counter and mix the bag before the second draw. Calculate the probability of exactly one red and compare it with the without-replacement probability.

S03.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Conduct ten two-draw trials under each model and record each ordered color pair in the response space below, labeling each model. Before every new trial, restore the bag to 4 red and 6 blue. Calculate the fraction with exactly one red for each model and compare it with the calculated probability. If live results are unavailable, use these demonstration results instead: 5 qualifying trials out of 10 without replacement and 4 out of 10 with replacement. State which results you used.

Discuss your results

- Why must the bag be reset between pairs even in the without-replacement experiment?
- Why can two small simulated fractions differ from their model probabilities?

INDEPENDENT APPLICATION

1. A standard 52-card deck contains 12 face cards and 40 other cards. Select two cards at random without replacement. Calculate the probability that exactly one is a face card. Given that the first is a face card, calculate the probability that the second is not a face card.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 3.5, Try It 3.25, parts a-b, printed p. 192. Adapted for this course; source data retained unless otherwise stated.

S03.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. Two independent deliveries each have late probability .10. Find P(at least one late).

3. A printer's first-page jam probability is .03. Given no first-page jam, second-page jam probability is .04. Find P(no first-page jam and second-page jam).

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

An event occurs through two disjoint paths with probabilities .12 and .08. Another student multiplies them. Explain the correct operation.

S03.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR17 for S04.01 and answer both checks.

1. A jar has 5 black and 3 white counters. For two draws without replacement, find probabilities of two black, two white, and one of each. Check the total.

2. Repeat the jar calculations with replacement and explain what changes.

3. In an illustrative model, 20% of packages are express. Express packages arrive late with probability .02; others with probability .10. Find P(late).

S03.04-HW | Continued

4. Using the package model, find P(express given late) and explain why it differs from P(late given express).

5. Recall: If independent events have P(A)=.25 and P(B)=.40, find their intersection, union, and the chance neither occurs.

6. A student multiplies (.3)(.3) for two red draws from a bag with 3 red and 7 blue without replacement. Identify the assumption and correct the result.

Unit 3 | Cumulative synthesis

1. Define one simulation trial for the chance of at least two successes in four independent attempts with success probability .30.

2. P(A)=.45, P(B)=.35, and P(A and B)=.15. Find P(A or B), P(neither), and P(A given B).

3. Can the events in the previous question be independent or disjoint?

4. From 4 green and 5 purple counters, find P(one of each) in two draws without replacement.

5. A simulation gives 9 extreme results in 3,000 repetitions under a claim. Interpret this and give one limitation.

6. Explain why equal observed counts in two differently sized groups do not establish independence.

Unit 04 | Random Variables

A chance model becomes more useful when its outcomes are organized as a numerical random variable. Students construct discrete and cumulative distributions, interpret probability-weighted averages, and measure variability. They then study two major models: the binomial distribution for counts and the normal distribution for continuous measurements. Conditions are part of each model, not an afterthought. Technology supports exact probabilities and inverse calculations, while sketches, event notation, and contextual interpretation make the work reviewable. The examples use illustrative model values. Geometric distributions and a standalone general unit on combinations of random variables are outside this core sequence. Unit 5 explicitly teaches the means and standard deviations of differences of independent sample proportions and sample means, including why their variances add.

CED topics: 2.8, 2.9, 2.10, 2.11

IN THIS UNIT

Lesson	Focus
S04.01	Random variables, distributions, and expected value
S04.02	The binomial model
S04.03	Continuous models and the normal distribution
S04.04	Normal probabilities, percentiles, and cutoffs

Count arrivals during equal observation windows

Window	Hikers who pass
9:00 to 9:30	0
9:30 to 10:00	3
10:00 to 10:30	1

Illustrative counts. A window with zero arrivals belongs in the record.

LOCAL QUESTION

Suppose X counts hikers who pass a fixed trail point during one half hour. What values can X take? What would you need to know before assigning probabilities to those values?

S04.01 | Random variables, distributions, and expected value

CED 2.8, 2.9

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Make a model from a random device

Let X be the number of delayed deliveries on one route. A model assigns probability 0.10 to X = 0, 0.30 to X = 1, 0.40 to X = 2, and 0.20 to X = 3. Simulate one route by generating one uniformly random integer from 0 through 9. Code 0 as X = 0, 1-3 as X = 1, 4-7 as X = 2, and 8-9 as X = 3. Each new digit must be independent.

Materials. Random-integer generator or ten equally sized slips replaced after each draw.

Complete the probability and cumulative tables

Digit device: each integer 0–9 is equally likely.

X	Digits	P(X = x)	P(X ≤ x)	x × P(X=x)
0	0			
1	1–3			
2	4–7			
3	8–9			

Check probability total: _____ Expected value: _____

The digit assignment is supplied by the investigation. Calculate the probability weights, cumulative values, and weighted contributions.

1. Complete the supplied organizer. For each value of X, enter P(X = x), the cumulative probability P(X ≤ x), and xP(X = x). Add the weighted contributions to obtain the model mean.

2. Simulate 20 routes and record all 20 values of X below. Calculate their sample mean. If your teacher supplies the backup dataset instead, use only that dataset and label your work “supplied simulation.”

3. Calculate the theoretical expected value and compare it with the 20-value mean.

S04.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Construct a second distribution on 0, 1, 2, 3 with all probability concentrated at 1 and 2, giving P(1)=.30 and P(2)=.70. Compare its mean and standard deviation with the first model.

Discuss your results

- Why can the sample mean differ from the expected value?
- Does equal expected value mean equal predictability?

INDEPENDENT APPLICATION

1. Two stacks each contain cards numbered 1 through 8. One card is selected at random from each stack, independently. Let X be the number of selected cards showing an odd number. Construct the probability distribution of X, then calculate its mean and standard deviation.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 4.2, Try It 4.7, printed pp. 236 (PDF pp. 248).

2. A proposed probability distribution assigns probabilities 0.20, 0.40, and 0.50 to its only three possible values. Is this a valid probability distribution? Justify your answer.

3. A delivery manager says a mean of 1.7 late packages predicts exactly 1.7 late packages tomorrow. Correct the interpretation.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

X takes only the values 0, 1, and 2. A cumulative table gives P(X at most 0)=.20, P(X at most 1)=.65, and P(X at most 2)=1. Find P(X=1) and P(X at least 1).

S04.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR18 for S04.02 and answer both checks.

1. A customer orders 1, 2, or 3 bottles of water with probabilities 0.60, 0.30, and 0.10, respectively. Let X be the number ordered. Construct a cumulative probability table and calculate P(X > 1).

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 4.2, Try It 4.3, printed pp. 231 (PDF pp. 243). Cumulative-probability task added.

Draw your table here. Include headings and all rows needed for the data.

2. For the water-bottle model in Question 1, calculate the mean, variance, and standard deviation of X. Interpret the mean in context.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 4.2, Try It 4.3, printed pp. 231 (PDF pp. 243). Variance and SD extend the expected-value question.

3. A net return X is 5 dollars with probability .10, 0 dollars with probability .30, and -2 dollars with probability .60. Find E(X) and interpret its sign.

S04.01-HW | Continued

4. X takes only the values 0, 1, 2, and 3. Its cumulative probabilities are .15, .55, .85, and 1 at those values, respectively. Recover the individual probabilities and calculate P(X at least 2).

5. Review. For independent events with probabilities .40 and .50, calculate the probability exactly one occurs.

6. A student averages 0, 1, 2 and reports mean 1 for probabilities .20, .50, .30. Explain the error and state when that method would work.

S04.02 | The binomial model

CED 2.10

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Count successes, then compare models

In a model, each seed has probability 0.30 of germinating, independently of all other seeds. Let X be the number that germinate in one set of eight seeds. One simulation repetition must represent all eight seeds and produce one value of X. The table above the questions records how often each count occurs across 20 complete sets.

Materials. Random digits 0-9 or a random-integer generator.

Tally the simulated success counts

Investigation model: n = 8 attempts, p = .30 per attempt.

X	0	1	2	3	4	5	6	7	8
Count									
Fraction									

Total simulated sets: _____

Event to estimate: _____

Qualifying sets / total sets: _____

Record one count X for each complete eight-attempt set. The frequency and relative-frequency rows are intentionally blank.

1. Describe how to simulate one set of eight seeds using uniformly generated digits 0-9. State which digits represent germination, how many digits make one set, and what value you record for the set.

S04.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Simulate 20 complete sets. In the organizer, enter the frequency of each count X = 0, 1, ..., 8 and its relative frequency. Verify that the frequencies total 20 and the relative frequencies total 1. If your teacher supplies backup frequencies, use those instead and label the table “supplied simulation.”

3. Use the table to estimate $P(X \geq 3)$. State the numerator and denominator, then calculate the exact binomial probability and compare the two results.

S04.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Calculate the theoretical mean and standard deviation of X. Suppose a different procedure tests seeds until three have germinated. Explain why the number of seeds tested in that procedure is not a binomial random variable.

Discuss your results

• Why might a possible value of X have frequency zero in only 20 simulated sets?

• Would a binomial model remain appropriate if the germination probability changed between seeds? Explain.

INDEPENDENT APPLICATION

1. A fair coin is tossed ten times independently. Define a binomial random variable for the number of heads. State its parameters and calculate its mean and standard deviation.

S04.02 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. A fair six-sided die is rolled ten times independently. Let X be the number of rolls showing a one. Calculate P(X > 3). Show the event and the binomial parameters used in your calculation.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 4.3, Try It 4.11, printed pp. 238 (PDF pp. 250). Completes the probability calculation requested symbolically in the source.

3. A school committee has ten staff members and six students. Two different members are selected at random without replacement. Let X be the number of students selected. Explain why an exact binomial model is inappropriate.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 4.3, Example 4.16, printed pp. 242 (PDF pp. 254). Committee wording lightly localized, counts unchanged.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

For a binomial X, explain how to obtain P(X greater than 5) from a cumulative probability.

S04.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR19 for S04.03 and answer both checks.

1. Each of eight seeds germinates with probability 0.30, independently of the others. Let X be the number that germinate. Explain why X has a binomial distribution, and calculate its mean and standard deviation.

2. For that seed model, calculate P(X=2) and P(X at least 3). Show the event translation.

3. For X binomial with n=20, p=.10, calculate P(X=0) and P(X at least 1). Interpret both.

S04.02-HW | Continued

4. A file contains 5,000 records, exactly 20% of which carry a particular label. A simple random sample of 50 records is selected without replacement. Let X count sampled records with the label. Explain why a binomial approximation may be reasonable, and state its parameters.

5. A discrete variable takes values 0, 1, and 2 with probabilities 9/16, 6/16, and 1/16. Calculate the expected value using the probabilities, then identify a binomial model with this distribution.

6. A student computes P(X at least 4) as 1-P(X at most 4). Explain precisely which probability was omitted.

S04.03 | Continuous models and the normal distribution

CED 2.11

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

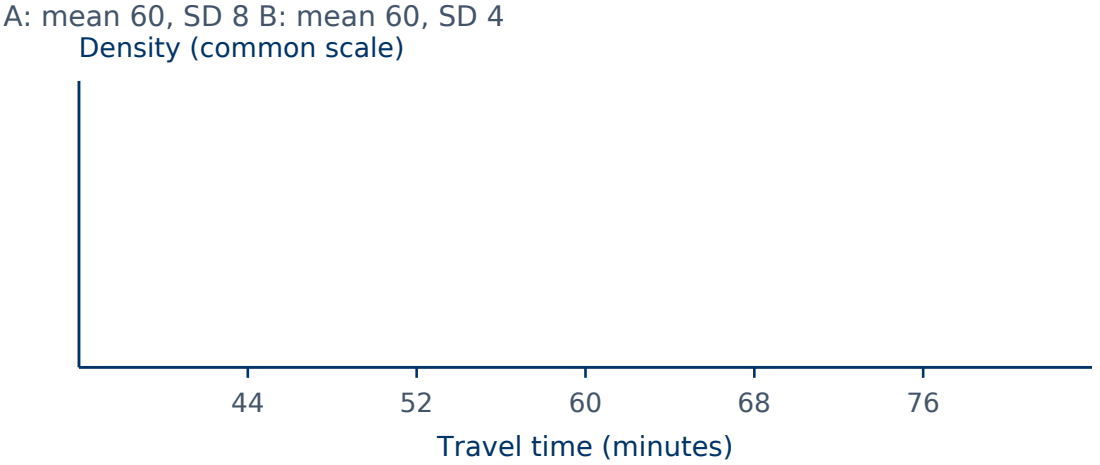
Same center, different spread

Let X be travel time in minutes. Compare two illustrative normal models: A has mean 60 minutes and standard deviation 8 minutes; B has mean 60 minutes and standard deviation 4 minutes.

Materials. Graphing technology or graph paper.

Compare center and spread graphically

Sketch both models on the same axes.



Label each curve and mark its central one-standard-deviation interval.

The axes share one scale. Curves and probability regions are left blank for the investigation.

1. On the supplied common axes, sketch and label both normal curves. Use travel time in minutes on the horizontal axis and density on the vertical axis. Label the shared mean and the endpoints one standard deviation from each model's mean.

2. Below the graph, write each model's central approximately 68% interval and its width. Explain why Model B's interval is narrower.

3. Estimate P(X>68) in each model with the empirical rule.

S04.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. A third travel-time distribution has mean 60 minutes and standard deviation 8 minutes but is strongly right-skewed. Would the same z-score calculation and the same empirical-rule tail estimates still be valid? Explain separately for each.

Discuss your results

- Why must the narrower curve be taller?
- What does a z-score of 1 say without assuming normality?

INDEPENDENT APPLICATION

1. Scores on a college entrance exam are approximately normally distributed with mean 52 points and standard deviation 11 points. Identify the central intervals containing approximately 68%, 95%, and 99.7% of scores.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.1, Try It 6.6, printed pp. 340 (PDF pp. 352).

2. For the exam-score model in Question 1, calculate the z-score for a score of 41 points and interpret it in context.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.1, Try It 6.6, printed pp. 340 (PDF pp. 352). Standardized-score interpretation added.

3. A normal density curve has height .08 at x=60. Is P(X=60)=.08? Explain.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A variable is strongly skewed. A student standardizes it and applies the empirical rule. Identify the problem.

S04.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR20 for S04.04 and answer both checks.

1. A normal model has mean 40 and standard deviation 5. Give the approximate central 68%, 95%, and 99.7% intervals.

2. Using that model, estimate P(X>50) and P(35<X<50) with the empirical rule.

3. Calculate the z-score for x=32 under N(40, 5). Find x corresponding to z=1.6 under the same model.

S04.03-HW | Continued

4. A continuous variable is recorded to the nearest minute. Explain why the recorded value 40 can occur even though the model gives P(X=40)=0.

5. Review. For a binomial variable with n=25, p=.20, calculate mean and standard deviation. Explain why the variable is discrete.

6. Two distributions have equal mean and standard deviation, but one is normal and one strongly skewed. Must the proportion above $\mu+2\sigma$ be the same? Explain.

S04.04 | Normal probabilities, percentiles, and cutoffs

CED 2.11

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Boundary or area?

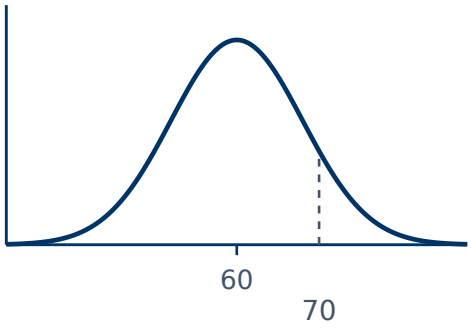
Let X be travel time in minutes. Use an illustrative normal model with mean 60 minutes and standard deviation 8 minutes, written N(60, 8). In this notation, the second parameter is the standard deviation. Work with a partner, but each student records inequalities.

Materials. Normal probability and inverse-normal technology, or a standard-normal table.

Shade the event and label the boundary

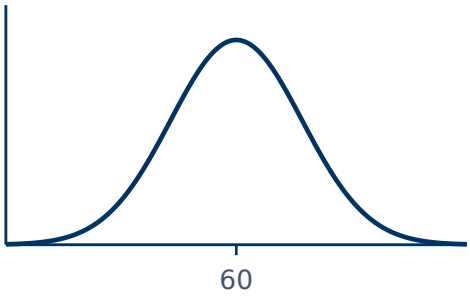
Use the illustrative N(60,8) model.

Find P(X > 70)



P(X > 70): _____

Find the top 10% cutoff



Left-tail area: _____

The first boundary is supplied; the second is unknown. Add shading, areas, and calculations to complete the investigation.

1. Use the first supplied normal curve for P(X > 70). Shade the appropriate region, label the boundary 70 minutes, and calculate the probability. In the lines below, show the standardized boundary and the left-tail area you subtract from 1.

S04.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Use the second supplied curve to find the cutoff c for the highest 10% of travel times. Label the 0.10 right-tail area and c. Calculate c, then verify that $P(X > c)$ is approximately 0.10.

3. Draw a separate normal curve below this question. Shade the central 80%, label both excluded tail areas, and calculate the two boundary times. Explain which boundary equals the cutoff from Question 2.

S04.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Compare a 70-minute trip from the N(60, 8) route with an 82-minute trip from a different normal route having mean 70 minutes and standard deviation 6 minutes. Use z-scores and percentiles.

Discuss your results

- What information switches roles in a forward and inverse problem?
- Does the trip with the higher percentile have the more desirable outcome?

INDEPENDENT APPLICATION

1. Golf scores for a school team are modeled by a normal distribution with mean 68 strokes and standard deviation 3 strokes. Calculate the probability that a randomly selected round has a score below 65 strokes. Show your setup.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.2, Try It 6.8, printed pp. 343 (PDF pp. 355).

S04.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. For the golf-score model in Question 1, find the 90th percentile. Interpret it in context.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.2, Try It 6.8 model and Example 6.8(c) method, printed pp. 341-343 (PDF pp. 353-355).

3. A cutoff marks the lowest 20%. Which cumulative probability should inverse-normal use? Explain.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

For a central 95% normal interval, name the two percentiles used and explain why.

S04.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR21 for C2 and answer both checks.

1. For the school golf-score model with mean 68 strokes and standard deviation 3 strokes, calculate $P(X > 65)$ and $P(66 < X < 70)$. Show the standardized boundaries and interpret the second probability.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.2, Try It 6.8 and Try It 6.9, printed pp. 343-344 (PDF pp. 355-356).

2. For the golf-score model in Question 1, find the cutoff for the lowest 10% of scores. Explain why the standardized cutoff is negative.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.2, Try It 6.8 model and Example 6.8(c) method, printed pp. 341-343 (PDF pp. 353-355). Percentile changed to the lower 10%.

3. For the golf-score model in Question 1, find the central 90% interval and state the probability in each excluded tail.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 6.2, Try It 6.8 model, printed pp. 343 (PDF pp. 355). Central-interval extension.

S04.04-HW | Continued

4. Student A scores 84 in a normal distribution with mean 72 and standard deviation 6. Student B scores 90 in a normal distribution with mean 83 and standard deviation 5. Compare relative standing using both z-scores.

5. Review. For independent events with probabilities .2 and .3, calculate the probability at least one occurs and explain the complement route.

6. A student uses inverse-normal(.05) to find the top 5% cutoff and reports a value below the mean. Correct the setup and give the standard-normal cutoff.

Unit 4 | Cumulative synthesis

1. A discrete distribution assigns probabilities .1, .6, .3 to values 0, 1, 2. Find its mean and P(X≥1).

2. For that distribution, find variance and SD.

3. Explain why counting attempts until two successes is not a binomial count.

4. For X binomial with n=10, p=.2, find P(X≥1), mean, and SD.

5. A population of test scores is modeled by a normal distribution with mean 80 points and standard deviation 12 points. Let X be the score of a randomly selected student. Calculate P(X > 104) and interpret it in context.

6. For the score model in Question 5, calculate the central 80% interval. Identify the percentiles used and give endpoints in points.

Unit 05 | Sampling Distributions

Inference depends on understanding how statistics change across repeated samples. This unit separates individual data from sample summaries, distinguishes bias from variability, and makes repeated sampling visible through small exact examples and simulation. It also introduces randomization distributions, which hold observed responses fixed while reallocating treatment labels under a no-effect model. Students then develop sampling models for proportions, means, and independent differences. Every calculation identifies the statistic and checks design, independence, and shape separately. These models lead directly to confidence intervals and hypothesis tests, where unknown population quantities must be estimated or specified by a null hypothesis.

CED topics: 2.12, 3.1, 3.2, 3.9, 4.1, 4.6

IN THIS UNIT

Lesson	Focus
S05.01	Sampling distributions and randomization distributions
S05.02	Sampling distributions of proportions and their differences
S05.03	Sample means, differences in means, and the CLT

A value for a user and a value for a sample

What produces one value?	Example
One trail-user interview	Resident: yes
One sample of 20 users	9 residents out of 20, or 0.45

The observations in a sampling distribution are statistics from repeated samples.

> **LOCAL QUESTION**
>
> If different groups sample trail users, how will their estimates vary? Distinguish the trail-user data from the distribution of sample statistics.

S05.01 | Sampling distributions and randomization distributions

CED 2.12, 3.1

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Shuffle assignments under no effect

Four seedlings in a classroom experiment have measured growth values of 2, 4, 8, and 10 centimeters. Two seedlings were randomly assigned to treatment and two to control, with all six allocations equally likely. The observed treatment seedlings grew 8 and 10 centimeters; the controls grew 2 and 4. To investigate a no-treatment-effect model, assume each seedling would have had the same growth under either assignment. Keep these four measured values fixed.

Materials. Four cards labeled 2, 4, 8, 10; paper for listing allocations.

Enumerate the reallocations

Fixed responses: 2, 4, 8, 10. Assign two to each group.
Observed assignment: treatment 8,10; control 2,4.

Trial	Treatment	Control	Mean T	Mean C	T − C
1					
2					
3					
4					
5					
6					

List each distinct allocation once. Keep response values fixed under no effect.

The observed responses and assignment are supplied information. Group allocations, means, and differences are left for students to determine.

1. Define D as treatment mean growth minus control mean growth, in centimeters. Calculate the observed value of D and record it below.

S05.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Complete the six rows of the supplied allocation table. In each row, list a different pair assigned to treatment and put the remaining pair in control. Enter both group means and D. Use each treatment pair once.

3. Use the six equally likely D values to describe the center and range of the exact randomization distribution. Include units.

S05.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Calculate the proportion of allocations with D at least as large as the observed D. State the numerator and denominator. Explain why the simulation changes group assignments but keeps each seedling's measured growth fixed.

Discuss your results

- Would collecting new response values be the same simulation?
- What feature of a real study would justify this random-assignment comparison?

INDEPENDENT APPLICATION

1. In the five-die experiment, a class records 40 sample means, each calculated from a different set of five independent die rolls. What does one plotted value represent? State the sample size and the number of simulation repetitions.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 7 Introduction, Collaborative Exercise, printed pp. 365 (PDF pp. 377). Number of classroom repetitions made explicit.

S05.01 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. An estimator's sampling distribution is centered at 12 when the true parameter is 10. Describe its bias and whether more repetitions remove it.

3. A class can either increase the number of dice per set from 5 to 20 or keep 5 dice per set and increase the number of sets from 40 to 160. Explain how each change affects its investigation of the sampling distribution of the mean.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 7 Introduction, Collaborative Exercise, printed pp. 365 (PDF pp. 377). Comparison of sample size and repetitions added.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A simulation repeatedly shuffles 20 observed responses into groups of 10 and records a difference in means. What distribution is being approximated?

S05.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR23 for S05.02 and answer both checks.

1. A population contains 1 and 5 equally often. Enumerate ordered samples of size 2 with replacement and their means. Calculate the mean of the sampling distribution.

2. For the population in Question 1, compare the population standard deviation and the standard deviation of the sample means.

3. An estimator takes values 6, 10, and 12 with probabilities 0.20, 0.40, and 0.40. For a population parameter of 10, determine whether the estimator is unbiased. Show how you found the center of its sampling distribution.

S05.01-HW | Continued

4. A volunteer survey gives stable estimates across many large samples. Does stability establish accuracy? Explain using bias and variability.

5. Review. A sample of size 2 is drawn with replacement from the values 2, 4, 8, each equally likely on every draw. Explain why all nine ordered samples are equally likely, but the six distinct resulting sample means are not.

6. An experiment has observed responses 1, 3, 5, 7, with 5 and 7 assigned to treatment. List all treatment-minus-control mean differences under equal-size reallocations and find the fraction at least as large as observed.

S05.02 | Sampling distributions of proportions and their differences

CED 3.2, 3.9

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Hands-on option. Use this investigation or "Bead draws | How much can a sample proportion vary?," then continue to Independent application.

Two populations, one difference

Suppose 60% of the 2,000 students in one population and 45% of the 2,000 students in a second population support a proposed after-school bus. Independently select a simple random sample of 100 from the first population and 120 from the second. Let p-hat 1 and p-hat 2 be the sample support proportions. These are hypothetical model values.

Materials. Calculator with normal probabilities; paper to record all four expected counts.

Check each group separately

Independent SRSs: compare sample proportion 1 minus 2.

Given model	Group 1	Group 2
Population proportion p	.60	.45
Sample size n	100	120
Population size N	2,000	2,000
10% check		
Expected successes		
Expected failures		

Difference center: _____ Difference SD: _____

The model parameters are supplied by the investigation. Complete both finite-population checks and all four expected counts before using a normal difference model.

1. Define D = p-hat 1 - p-hat 2 in words. Calculate and interpret the mean of its sampling distribution.

S05.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Complete the supplied condition organizer. Show each sample size as a fraction of its own population, then calculate expected supporters and nonsupporters in each group. State whether these checks justify the usual approximate normal model.

3. Calculate the sampling standard deviation of D and approximate $P(D > 0.25)$. Show the two variance contributions, the standardized boundary, and the probability. Interpret the result in context.

4. Now define the difference in the reverse order, p-hat 2 - p-hat 1. State its mean and SD. Write the inequality for the reversed difference that describes the same event as $D > 0.25$.

Discuss your results

- Why do the variance terms add?
- What fails if the same 120 people are surveyed before and after a program?

S05.02 | Optional lab | Bead draws | How much can a sample proportion vary?

Optional guided application | 20 minutes

Compare the variation in sample proportions from samples of 10 and 30 draws when the target-color proportion stays at 0.40.

Materials. One opaque bag of 100 equal-size, equal-weight beads per team, with exactly 40 target-colored and 60 other beads. Provide a tally sheet and two class dotplots on the same 0-1 scale.

COLLECT YOUR DATA

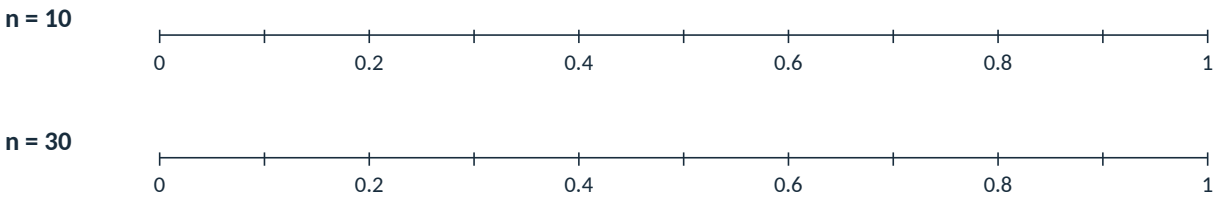
1. Mix the bag. Without looking, draw one bead, record whether it has the target color, return it, and mix again. Repeat for 30 draws. Pause after draw 10 to record the target count.

2. Record the total target count through draw 30. Divide the first count by 10 and the total by 30. Each team posts one dot on the n = 10 class plot and one on the n = 30 plot.

3. Compare the centers and spreads on the same horizontal scale. Identify the meaning of one dot on each plot. Use SD = $\sqrt{[p(1 - p)/n]}$ to find the predicted spread at each sample size.

Team	Targets in first 10	Targets in all 30	Proportion in first 10	Proportion in all 30
Our team				
Another team				

Class dotplots | Each dot is one team's target-color proportion.



ANALYZE YOUR RESULTS

1. Does the n = 30 plot look less spread out? Compare the two theoretical SDs with the class plots, and explain a discrepancy if you see one.

2. A student says that every team's proportion must get closer to 0.40 after 30 draws. Write what the larger sample size actually lets you predict for repeated samples from this bead bag.

Continue to this lesson's Independent application.

S05.02 | Practice and exit

INDEPENDENT APPLICATION

1. Use the OpenStax school-homework model with independent completion outcomes and p = 0.70, now for 100 students. Calculate the mean and standard deviation of the sample proportion who complete on time. Verify the large-count condition.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 4.3, Example 4.12, printed pp. 238-239 (PDF pp. 250-251). Sample size increased to compare sampling variability.

2. If n rises from 100 to 400 with p fixed, what happens to the sampling standard deviation?

3. For n=100, p=.02, explain why a normal model for p-hat is doubtful.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Independent random samples of 100 and 200 come from large populations with proportions $p_1 = 0.50$ and $p_2 = 0.20$. State the sampling mean and SD of $\hat{p}_1 - \hat{p}_2$. Show the variance contribution from each population.

S05.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR24 for S05.03 and answer both checks.

1. Suppose 30% of the 10,000 members of a district student population participate in community service. A simple random sample of 100 students is selected without replacement. Let p-hat be the sample proportion who participate. Describe the shape, mean, and standard deviation of its sampling distribution. Verify the necessary conditions.

2. Using Question 1, approximate P(p-hat>.40) and interpret it.

3. For p=.30, compare sampling standard deviations at n=100 and n=400. How much must n increase to reduce standard deviation to one third of its original size?

S05.02-HW | Continued

4. Two district populations of 10,000 students have community-service participation proportions p1 = 0.50 and p2 = 0.40. Select independent simple random samples of 100 from each. Approximate the probability that the first sample's participation proportion is lower than the second's. Verify conditions and show your calculation.

5. Review. Explain the difference between a sampling distribution centered away from p and a single p-hat that differs from p.

6. A student checks $n1+n2 \le 10\%$ of one population and only the combined success count for two independent samples. Correct both checks.

S05.03 | Sample means, differences in means, and the CLT

CED 2.12, 4.1, 4.6

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

A skewed population, more stable averages

A model population consists of five equally likely cards labeled 0, 0, 0, 0, and 10. These numbers represent an abstract quantitative variable, not measured class data. Draw with replacement so the draws are independent. The population mean is 2 and its standard deviation is 4.

Materials. Five cards or independent random integers 1-5, coding 1-4 as 0 and 5 as 10; calculator.

Keep observations and sample means separate

Population cards: 0, 0, 0, 0, 10.

Replace after every draw. Record one mean per sample of five.

Sample	Draw 1	Draw 2	Draw 3	Draw 4	Draw 5	Mean
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

Use the individual-draw columns to calculate one sample mean in the final column. Pool the means to build a sampling-distribution display.

1. Describe the shape of the population distribution. Verify the stated mean 2 and standard deviation 4 using the five equally likely card values.

2. Complete the ten-row organizer. For each row, draw five times with replacement, record the five values, and calculate one sample mean. Pool only the sample-mean column with the class. If your teacher supplies backup means instead, label them as supplied values and leave the individual-draw columns blank because those observations were not supplied.

S05.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. Calculate the theoretical mean and standard deviation of the sampling distribution for each sample size n = 1, 5, and 100. Explain which feature changes as n increases.

4. Explain what a large simulation of means for n=100 should look like compared with the original population, and what remains unchanged.

Discuss your results

- Does observing one roughly symmetric sample prove the CLT?
- How would you change the procedure to simulate a paired before-after mean difference?

INDEPENDENT APPLICATION

1. A population has mean 45 and standard deviation 8. For independent random samples of size 30, calculate the mean and standard deviation of the sampling distribution of the sample mean. Does this calculation alone establish an exactly normal sampling distribution? Explain.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 7.1, Try It 7.1, printed pp. 368 (PDF pp. 380). Restricts the source's unknown-shape example to justified center/spread calculations and an explicit shape question.

2. A strongly skewed population is sampled with n=4. Does a known mean and standard deviation justify a normal probability calculation for its average?

3. Twenty people provide before-and-after scores. Identify the quantitative variable for a paired analysis and the sample size.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

What changes when n is multiplied by 4: the population SD, the sampling mean, or the sampling SD of the mean?

S05.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR25 for S06.01 and answer both checks.

1. A normal population has mean 100 and standard deviation 16. For an SRS of 64 from 10,000, find P(sample mean<97) and justify the model.

2. Compare P(X<97) for one individual from that population with the sample-mean probability. Explain why they differ.

3. Two independent normal populations of travel times have means 12 and 10 minutes and standard deviations 3 and 6 minutes. Independent simple random samples of 36 and 64 trips are each less than 10% of their respective populations. Let D be the first sample mean minus the second. Calculate the mean and standard deviation of D, then approximate P(D > 3). Interpret the probability.

S05.03-HW | Continued

4. One sample has n1=10 from a strongly skewed population; another has n2=100. Explain why total n=110 is not enough to justify a normal difference-of-means model.

5. Review. A population proportion is .50. For n=100, calculate the standard deviation of p-hat and compare how it changes when n becomes 400.

6. A report says, "With n=40, the original waiting times become normally distributed." Rewrite it accurately and include a limitation.

Unit 5 | Cumulative synthesis

1. Explain the difference between sampling more individuals per sample and generating more simulated samples.

2. In repeated independent samples of 150 students from a population with walking proportion p = 0.40, let p-hat be the sample walking proportion. Calculate its sampling mean and standard deviation, then verify the large-count condition.

3. Independent random samples of sizes 100 and 120 are drawn from two large populations with proportions p1 = 0.60 and p2 = 0.45. Treat draws within each sample as independent. Calculate the mean and standard deviation of p-hat 1 - p-hat 2.

4. A normal population of waiting times has mean 20 minutes and standard deviation 6 minutes. For nine independent randomly selected waits, calculate the probability that their sample mean exceeds 22 minutes. Explain why the normal sampling model is appropriate.

5. Explain why two measurements per person require a paired analysis rather than an independent-two-sample SD formula.

6. Describe a randomization distribution for a randomized experiment comparing two mean responses.

Unit 06 | Confidence Intervals for Proportions

A sample proportion gives a useful estimate, but another random sample would produce another value. This unit turns that sampling variability into an interval estimate. Students first investigate what confidence means across repeated samples, then connect standard error, critical value, and margin of error. They learn to justify conditions, choose a sample size, compare two population proportions, and communicate what an interval supports. Every conclusion must name the population and response being estimated. The calculations address random sampling variation; they cannot repair a biased sampling frame, leading question, or unrepresentative set of volunteers.

CED topics: 3.3, 3.4, 3.10, 3.11

IN THIS UNIT

Lesson	Focus
S06.01	What a confidence interval says
S06.02	Build and justify a one-proportion interval
S06.03	Plan precision before collecting data
S06.04	Estimate a difference in population proportions

Classify a site before estimating a proportion

Site	Temperature at noon (°C)	Above 18 °C?
A	17.8	No
B	18.0	No
C	18.2	Yes

Illustrative measurements. Exactly 18 °C is outside the “above 18 °C” category.

LOCAL QUESTION

For a specified sampling date and temperature threshold, what proportion of sites on a monitoring roster exceed the threshold? Define the measurement and how you would select the sites.

S06.01 | What a confidence interval says

CED 3.3, 3.4

OPENING PREDICTION

Two sample intervals are (0.29, 0.49) and (0.34, 0.54). Predict which crosses a line at the known rate 0.50. Sketch both to check. What changed between samples?

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Catch a known proportion

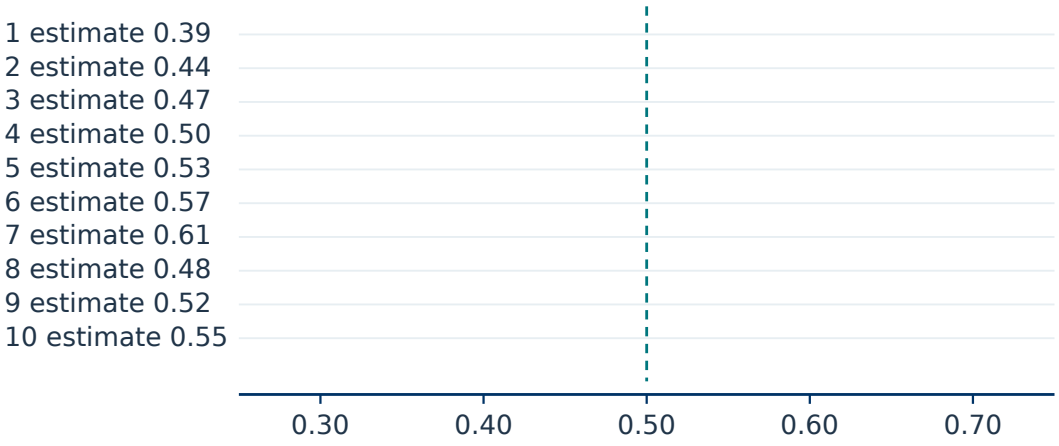
Use a population model with p = 0.50. In each trial, generate 100 independent zeros or ones with equally likely outcomes, calculate the sample proportion, and form the provisional interval estimate plus or minus 0.10. This simplified activity illustrates capture; the formal estimated-standard-error method comes next. If technology is unavailable, use these illustrative trial estimates: 0.39, 0.44, 0.47, 0.50, 0.53, 0.57, 0.61, 0.48, 0.52, 0.55.

Materials. Calculator or spreadsheet random-number generator; graph paper.

Investigation recording sheet

Construct the intervals; then check capture

Supplied-data route: use estimate ± 0.10 and compare with p = 0.50.



Captured: _____ / 10 One-sentence interpretation: _____

Use the displayed estimates for the supplied-data route. For live simulation, cross out each printed estimate and write the corresponding sample proportion; redraw the axis on separate paper if necessary. Draw ten interval segments and record their capture decisions.

1. Before simulating, predict whether every interval will contain 0.50 and explain your prediction.

S06.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Generate 10 independent samples of 100 binary outcomes and calculate one sample proportion per sample. If your teacher assigns the supplied example estimates instead, use those 10 values. Record which data source you used and list the lower and upper endpoints, estimate ± 0.10, for each sample.

3. On graph paper, draw the 10 intervals as separate horizontal segments sharing one proportion scale. Mark the fixed population proportion 0.50 with a vertical line. Count how many segments contain 0.50, including a segment with an endpoint at 0.50. Use the recording sheet for the endpoint and capture entries.

S06.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Pool the class results. If one group captures p in 8 of 10 trials and another in 18 of 20, what fraction of all 30 intervals captures p?

5. For a fresh sample estimate of 0.57, compare margins 0.05 and 0.15. Explain what increased width buys and what it costs.

Discuss your results

- What changed across repetitions: the parameter, the estimate, or both?
- Why would eight captures in ten repetitions not establish that a method's long-run capture rate is 80%?

S06.01 | Practice and exit

INDEPENDENT APPLICATION

1. A 90% confidence interval for the proportion of all St. Mary's School families favoring a proposed bus service is (0.42, 0.58). (a) Determine the point estimate and margin of error. (b) Does the interval provide convincing evidence that a majority of families favor the service? Justify your answer.

2. A random sample of 200 district households yields a 95% interval of 60% to 72% for support for a school levy. Rewrite this claim: "There is a 95% chance that 95% of our respondents support the levy."

3. A voluntary website poll reports 80% approval with a tiny margin of error. Explain why a much smaller random sample may provide a more defensible population estimate.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A 95% confidence interval for the proportion of all St. Mary's families favoring a proposed bus route is (0.48, 0.56). Does this interval provide convincing evidence that a majority favor the route? Justify your answer.

S06.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR26 for S06.02 and answer both checks.

1. An estimate is 0.64 with margin 0.055. Find both endpoints and the total width. Express the margin in percentage points.

2. A textbook survey of university students reports a 90% confidence interval of (0.564, 0.636) for the proportion registered to vote. Assume the students were randomly sampled from the university and the interval conditions are satisfied. (a) Interpret the confidence interval. (b) Interpret the 90% confidence level. Use separate sentences.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 8.3, Example 8.11, printed p. 423 (PDF p. 435). Source interval and context retained. Sampling assumptions are stated explicitly, and the task separates interval from level.

3. Three studies report intervals (0.32,0.49), (0.48,0.55), and (0.54,0.63). Which support a majority, which support less than half, and which are inconclusive about the half-way threshold?

S06.01-HW | Continued

4. A student says that a 99% interval is more precise than a 90% interval from the same data. Explain the error without doing any calculations.

5. Cumulative: In an SRS of 80 students, 48 report walking to school. Identify the statistic and parameter. Is the parameter known from this sample?

6. An organization computes 100 independent intervals using a nominal 95% method. Must exactly five miss the population proportion? Explain. Then explain why a reported 95% interval does not account automatically for nonresponse bias.

S06.02 | Build and justify a one-proportion interval

CED 3.3, 3.4

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Hands-on option. Use this investigation or "Globe toss lab," then continue to Independent application.

Four surveys, four diagnoses

Four illustrative surveys ask whether households compost. **Materials.** Calculator; four study cards.

Audit each survey before calculating

Survey and given information	Condition evidence to record	Usual z interval justified?
A: SRS; N = 6,000; n = 100; x = 62	Random: ____; fraction: ____; counts: ____	_____
B: voluntary website poll; n = 1,000; x = 620	Random: ____; fraction: ____; counts: ____	_____
C: SRS; N = 6,000; n = 30; x = 4	Random: ____; fraction: ____; counts: ____	_____
D: SRS; N = 500; n = 100; x = 62	Random: ____; fraction: ____; counts: ____	_____

Record a specific reason for every decision. The table supplies study facts, not diagnoses.

1. Use the survey-audit table to decide whether each of surveys A-D meets the conditions for the usual one-proportion z interval. Mark Yes or No for each study and give a specific reason.

2. For each survey, define p as the proportion of its target household population that composts. Record the observed number that compost and the number that do not in the counts space.

S06.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. In the table, identify the sampling method and calculate n/N when N is given. If the population size is not given, state that limitation. Explain separately whether random selection and the 10% condition are supported. Record the checks in the table.

4. For every justified study, compute the 95% interval and explain why equal sample proportions do not imply equal quality of evidence.

5. Redesign one unjustified study so that it has a defensible sampling basis and sufficient observed counts. Identify what cannot be guaranteed before collecting responses.

Discuss your results

- Why does C fail although its sample is under 10% of the population?
- Why would copying A's endpoints onto B or D hide an important problem?

S06.02 | Optional lab | Globe toss lab

Optional guided application | 20 minutes

Estimate the share of valid catches classified as water. Decide what would have to be true for that estimate to describe Earth's surface.

Materials. One inflatable Earth globe, Class tally and pencil, Calculator.

COLLECT AND CLASSIFY

1. Use short, gentle underhand tosses between willing partners in a clear space. Vary the starting orientation and give the globe a gentle spin. A seated student can record or read the marks.

2. On each catch, freeze the globe. Score one point at the center of the right index fingertip pad where it first touched the globe during the catch. Decide the spot before moving the finger.

3. Use the printed map at that point. Record clear ocean, lake, or other mapped water as W and clear land as L. Record an unresolved coastline, seam, or unreadable point as U. Keep every U in the tally.

4. If the globe drops or the right index finger misses it during the initial catch, mark R and retry. Give every valid catch one tally. Report W as water and L or U as nonwater under this rule.

1 | Define both proportions

Define the proportion this catching rule can estimate. What geographic proportion would you like it to represent?

RECORD THE CLASS DATA

Enter batch counts or tally marks. In the final row, combine all class results. Valid catches n = W + L + U. Report retries separately.

Batch	Water (W)	Land (L)	Uncertain (U)	Retry (R)	Valid (n)
1					
2					
3					
4					
5					
6					
Class total					

S06.02 | Analyze the globe tosses

ESTIMATE AND INTERPRET

Illustrative fallback data. A hypothetical round has 42 clear-water and 18 clear-land contacts in 60 valid catches, with no uncertain marks. Use these data separately if either live count is below 10. Keep the classroom result labeled as observed.

2 | Estimate and recode

Pool the counts from all groups. Find W/n. How would the estimate change if every U counted as water?

3 | Check and calculate

For your chosen dataset, report W, n − W, and the source. If both counts reach 10, calculate a model-based 95% z-interval for the catch-protocol proportion. State the independence assumption. Interpret both endpoints for valid catches under this rule.

4 | Compare and explain

After your teacher reveals the benchmark, compare it with your result. Name one feature of the map or the toss that limits an Earth-wide claim, and explain how it could matter.

Continue to this lesson's Independent application.

S06.02 | Practice and exit

INDEPENDENT APPLICATION

1. An SRS of 120 of 5,000 trail-pass holders finds 78 supporting an online renewal option. Construct a 95% interval.

2. An SRS of 300 voters contains 7 undecided voters. Is the ordinary one-proportion z-interval justified for the undecided proportion?

S06.02 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

3. The calculator reports (0.52,0.68). Write the additional information a reader needs to evaluate the inference.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

An SRS of 40 of a town's 4,000 households finds that 18 use a rain barrel. A report gives a benchmark proportion of 0.20. To construct a confidence interval for the current population proportion, which success and failure counts should you use to check the large-count condition? Explain.

S06.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 3, 4, 6. **Optional practice.** 2, 5. **Prepare for the next meeting.** Read PR27 for S06.03 and answer both checks.

1. A researcher randomly selects 250 people from a population of 10,000 adults. All respond, and 98 report owning a tablet. Construct and interpret a 95% confidence interval for the proportion of adults in this population who own a tablet. Show the method and verify its conditions.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 8.3, Try It 8.10, printed pp. 422-423 (PDF pp. 434-435). Counts and ownership context retained. A population size and complete response are supplied so all AP conditions can be evaluated.

2. An SRS of 300 of 20,000 customers contains 180 who would use paperless statements. Construct a 90% interval and assess whether at least 55% support is established.

3. For each case, name the failed condition: (a) SRS of 60 of 400 members with 42 successes; (b) 90 volunteers with 54 successes; (c) SRS of 60 of 4,000 members with 56 successes.

S06.02-HW | Continued

4. A student computes a 95% interval for 90 successes in 150 trials using SE = 0.60(0.40)/150. Identify the missing operation and give the corrected result.

5. Cumulative: A sample of 100 is drawn without replacement from 700 people. Does being an SRS make the 10% condition automatic? Explain and state the smallest N satisfying that condition.

6. An SRS gives a 95% interval (0.473,0.627) for a district proportion. A student concludes, Exactly 55% support the plan, and 95% of people agree. Correct both parts.

S06.03 | Plan precision before collecting data

CED 3.3, 3.4

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Buy a narrower interval

Use illustrative estimate p-hat=0.50. Compare sample sizes 100,400,900 and confidence levels 90%,95%,99%. Treat p-hat as fixed for this planning exercise.

Materials. Calculator or spreadsheet; blank three-by-three table.

Calculate the margin matrix

Confidence and supplied z*	n = 100	n = 400	n = 900
90%; z* = 1.645	_____	_____	_____
95%; z* = 1.960	_____	_____	_____
99%; z* = 2.576	_____	_____	_____

Use p-hat = 0.50 and MOE = z*sqrt [0.25/n]. Circle the smallest margin and explain one comparison in words.

1. Predict which combinations will give the narrowest and widest intervals.

2. Calculate the estimated standard error for each sample size: 100, 400, and 900. Show the three calculations below.

3. Complete all nine cells of the margin-of-error table using p-hat = 0.50 and the supplied critical values 1.645, 1.960, and 2.576. Enter each margin in proportion units. Circle the smallest margin.

4. Compare n=100 with n=400 at 95%. Compare 95% with 99% at n=400. Explain each change.

S06.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

5. A budget permits 400 completed responses. Decide whether a 99% margin at most 0.05 is possible under this conservative plan. Justify numerically.

Discuss your results

- Why do repeated samples not yield perfectly identical widths?
- What study defect would all nine narrow intervals leave unresolved?

INDEPENDENT APPLICATION

1. Find the conservative sample size for 95% confidence and margin 0.03. A population of 100,000 is available.

2. A pilot estimate is 0.70. For 95% confidence and margin 0.04, compare the planning size using the pilot with the conservative choice.

3. An interval's margin is 0.08 at n=150. Estimate n for margin 0.04 at the same confidence and proportion.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A planning calculation gives n >=384.15. A researcher chooses 384 and then doubles it to halve the margin. Identify both mistakes.

S06.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR28 for S06.04 and answer both checks.

1. An internet marketing company wants to estimate the proportion of customers who click on advertisements on their smartphones. How many completed responses are needed for 90% confidence and a margin of error no greater than five percentage points? No reliable prior estimate is available. Show your work and state the required sampling method.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 8.3, Try It 8.14, printed p. 427 (PDF p. 439). Source question and values retained; sampling requirements are requested explicitly.

2. A study needs 99% confidence and margin 0.04, with no prior estimate. Find n. Compare it with 601, the 95% requirement for the same margin.

3. At 95% confidence, a study with n=250 has estimated margin 0.06. Estimate required n for margin 0.03 and for margin 0.02, holding the proportion fixed.

S06.03-HW | Continued

4. A planned sample of 700 is drawn without replacement from 4,000 members. A student says the large n guarantees a valid ordinary interval. Explain the problem and what a careful plan should do.

5. Cumulative: A poll reports (0.54,0.66). Find its estimate, margin, and width. Does the interval support a population proportion of 0.50?

6. An analyst proposes interviewing the first 2,000 people entering one grocery store instead of randomly selecting 600 district residents. Critique the claim that the larger sample must be better, and identify a defensible use of the smaller sample.

S06.04 | Estimate a difference in population proportions

CED 3.10, 3.11

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Same table, opposite order

Use illustrative independent SRS data: service A 84 yes, 36 no; service B 66 yes, 54 no. Each service has at least 5,000 users.

Materials. Calculator; two group labels.

Build the two-proportion interval

Organize the two-group interval argument

Given: independent SRSs; each population has at least 5,000 users.

Service	Yes	No	n
A	84	36	120
B	66	54	120

Parameter in words: _____

Order: p____ − p____ Estimate: _____ SE: _____

Four observed counts: _____ Two 10% checks: _____

95% interval: _____ What does its sign support? _____

The observed table is given. Students must supply the parameter, calculation, conditions, and interpretation.

1. Let pA and pB be the proportions of all users of services A and B who favor online renewal. Calculate p-hat A − p-hat B. Record the parameter definitions and subtraction order in the organizer.

2. In the conditions space, identify the independent random samples, check the two sampling fractions, and list all four observed success and failure counts.

S06.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

3. In the calculation space, show the unpooled standard error and construct a 95% interval for pA – pB. Then interpret the interval in context using percentage points.

4. Reverse the group order and obtain the interval for pB - pA without recalculating SE.

5. A manager requires an advantage of at least 10 percentage points to justify a new program. Does this interval establish that threshold? Explain.

Discuss your results

• Why would pooling be inappropriate for this estimation problem?

• Why does a negative lower endpoint not by itself prove that service B has the greater proportion?

S06.04 | Practice and exit

INDEPENDENT APPLICATION

1. Independent random samples of 230 men and 218 women from two adult populations of at least 5,000 each find that 16 men and 18 women favor tea as their beverage. Construct and interpret a 95% confidence interval for pM − pW, where each parameter is the proportion in the corresponding population who favor tea.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 10.3, Try It 10.9, printed pp. 525-526 (PDF pp. 537-538). Original counts and beverage context retained. An interval replaces the source test, and independent random samples and population sizes are stated.

2. Let pNorth and pSouth be the proportions of all adults in the North and South districts, respectively, who support expanding bus service. A 95% confidence interval for pNorth − pSouth is (−0.09, −0.02). (a) Interpret the interval in context. (b) Give the corresponding interval for pSouth − pNorth.

S06.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

3. The same 100 students answer a yes/no question before and after a lesson. Explain why the ordinary two-independent-proportions interval is not justified.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A 95% difference interval is (-0.03,0.08). Does it prove the population proportions are equal?

S06.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 4, 6. **Optional practice.** 3, 5. **Prepare for the next meeting.** Read PR29 for C3 and answer both checks.

1. Independent SRSs of 250 service-A users and 220 service-B users, from populations of 10,000 users each, find that 150 service-A users and 110 service-B users favor electronic bills. Construct and interpret a 95% confidence interval for pA − pB, where each parameter is the proportion of all users of the corresponding service who favor electronic bills.

2. For an interval (-0.14,-0.04) for p1 - p2, state which group has the larger estimated proportion, give the reversed interval, and assess a claim of a 10-percentage-point difference.

3. A randomized experiment assigns 160 participants to A and 160 to B; outcomes are 96 and 64 successes. Compute a 95% interval for pA - pB. State the role of random assignment and whether a population 10% check is required by this assignment.

S06.04-HW | Continued

4. A student uses (84+66)/(120+120) in both parts of the interval standard error for the renewal data. Explain why that is inappropriate.

5. Cumulative: A class wants a one-proportion 95% interval with margin half as large as last time. If last time n=200, what sample size is planned under otherwise equal conditions, and what bias might remain?

6. A news report says, The difference interval is (0.02,0.09), so the first program increases success by between 2% and 9% for everyone. Rewrite the claim, assuming the study used independent observational random samples.

Unit 6 | Cumulative synthesis

1. Explain how a 95% confidence level differs from a 95% confidence interval.

2. An SRS of 120 of a district's 6,000 adults finds 72 favoring later library hours. Construct and interpret a 95% confidence interval for the population support proportion. Verify the conditions.

3. An analyst plans 95% confidence with margin 0.025 and no prior proportion estimate. Find n.

4. Why are separate observed proportions used in a confidence interval for p1-p2?

5. A difference interval (-0.02,0.07) is reported. Explain why neither equality nor a positive difference is established.

6. A survey's confidence interval is extremely narrow but half the sampled people did not respond. State the additional concern and why the margin does not resolve it.

Unit 07 | Hypothesis Tests for Proportions

A hypothesis test asks whether observed data are sufficiently unusual under a stated null model to support an alternative claim. This unit builds that logic with simulation before formal z procedures. Students test one proportion, compare two proportions, analyze errors and power, and investigate categorical tables with chi-square tests. The study design determines the scope of every conclusion. Statistical significance is evidence against a model, not proof of a claim or a measure of practical importance. Students practice defining parameters, checking the correct conditions, interpreting p-values, and connecting numerical evidence to a careful contextual conclusion.

CED topics: 3.5, 3.6, 3.7, 3.8, 3.12, 3.13, 3.14, 3.15

IN THIS UNIT

Lesson	Focus
S07.01	Hypotheses, null models, and p-values
S07.02	Carry out a one-proportion z test
S07.03	Errors, power, and consequences
S07.04	Test a difference in population proportions
S07.05	Compare categorical distributions with chi-square
S07.06	Investigate association with chi-square

Who could be missing from an archive?

Source	Limit of a claim
Surviving yearbooks from two eras	Printed names can omit students who were enrolled.
A complete contemporaneous roster	A roster supports a coverage check against the yearbook.

Record the number of unmatched names before comparing the eras.

LOCAL QUESTION

An archive holds student records from different eras. Which students could those records represent? What evidence would justify testing a claim about a broader population?

S07.01 | Hypotheses, null models, and p-values

CED 3.5, 3.6, 3.7

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Which results count as extreme?

Illustrative study: a binary process produces 15 successes in 20 independent trials. Investigators specified Ha: p > 0.50 before observing the outcomes. Simulate under p = 0.50. Fallback summary for 1,000 illustrative null trials: 23 counts are at least 15; 20 counts are at most 5.

Materials. Coins or a random-number generator; tally sheet.

Describe a reproducible null simulation

Component	Your procedure
Study to model	Observed 15 successes in 20 trials; H0: p = 0.50; Ha: p > 0.50
Random outcome rule	_____
One complete simulated sample (20 outcomes)	_____
Statistic recorded	_____
Repetitions and extremeness rule	_____
Estimated p-value	Qualifying repetitions / total repetitions = _____

Specify enough detail for another group to reproduce the simulation. One repetition must generate a full sample of 20 binary outcomes.

1. Complete the simulation organizer. State H_0 and H_a, specify how to generate one success or failure under H_0, define one complete simulated sample of 20 outcomes, and identify the statistic recorded for that sample.

2. Generate 20 independent outcomes per simulated sample and repeat for at least 20 samples per group. Pool the class's sample counts. If your teacher assigns the supplied example simulation results instead, use those and label them as supplied results.

3. Estimate the p-value for the stated greater-than alternative. Explain exactly which counts belong in the numerator.

S07.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. For a separately prespecified two-sided question about p not equal to 0.50, identify symmetric departures at least as far from 10 successes as the observed 15. Use the fallback counts to estimate its p-value.

5. Compare both estimated p-values with alpha = 0.05. Discuss why their decisions can agree or differ in other samples.

Discuss your results

- If a class produces no count at least 15, is that event impossible under the null?
- Why must the alternative come from the original question?

INDEPENDENT APPLICATION

1. A town asks whether the proportion of commuters cycling to work differs from 0.12. Define p and write hypotheses.

2. A test has p-value 0.18 and alpha 0.05. State the decision and explain why it does not prove H0.

3. A simulation models random samples of 50 residents under H_0: p = 0.30, where p is the proportion of all town residents favoring a proposal. In the actual sample, 22 of 50 favor it. Of 2,000 simulated samples, 47 contain at least 22 supporters. Estimate and interpret the p-value for H_a: p > 0.30.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A student interprets p-value 0.04 as a 4% probability that H0 is true. Rewrite the statement correctly.

S07.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR31 for S07.02 and answer both checks.

1. State appropriate null and alternative hypotheses for each question. Define the population parameter in each case. (a) Is the proportion of registered voters in Jackson County who voted in a specified primary election greater than 0.30? (b) Is the proportion of first-time applicants who pass an Oregon driver's test on the first attempt greater than 0.40?

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 9.1, Example 9.1 and Try It 9.4, printed pp. 462-463 (PDF pp. 474-475). Benchmarks retained; locations personalized to southern Oregon. Nulls use the equality form expected in AP work.

2. Describe a reproducible null simulation for an SRS of 50 people when H0: p = 0.20 and the observed number of successes is 16. The alternative is greater than.

3. In 5,000 null simulations, 185 statistics meet the prespecified extremeness rule. Find the estimated p-value and decisions for alpha 0.05 and 0.01.

S07.01-HW | Continued

4. A researcher first sees a positive sample difference, then decides to test only for a positive population difference. Explain the inferential problem.

5. Cumulative: An SRS of 100 from 3,000 members finds 60 supporters. Explain separately what random sampling and the 10% check contribute.

6. An analyst claims, My p-value is 0.27, so the null is correct and my experiment had a 27% chance of failing. Identify both errors and provide a useful report.

S07.02 | Carry out a one-proportion z test

CED 3.5, 3.6, 3.7

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

One result, three alternatives

Use the illustrative observed estimate 0.60 from n = 120, with null p0 = 0.50. The data come from an SRS without replacement of 120 members from a club of 6,000 members; 72 favor the proposed schedule. Consider three separate research questions that would have been chosen in advance: greater than, less than, and different from 0.50.

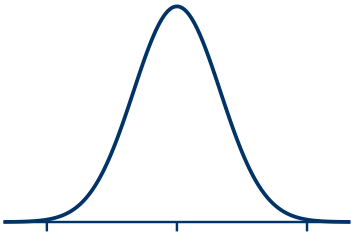
Materials. Calculator with normal probability function; sketch of standard normal curve.

Show the probability your test calculates

Shade the probability for each stated alternative

Given: x = 72, n = 120, p0 = 0.50. Use the same z in all three panels.

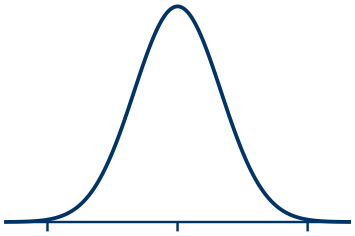
Ha: p > 0.50



p-value: _____

Right tail

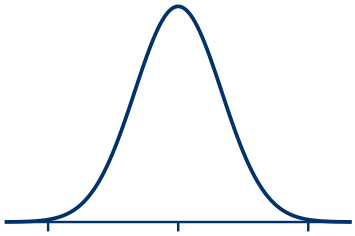
Ha: p < 0.50



p-value: _____

Left tail

Ha: p ≠ 0.50



p-value: _____

Both tails

Expected counts: _____ Shared test statistic z: _____

Choose the alternative before observing the sample. Mark z and label each shaded tail.

For task 2, mark the observed statistic and shade the relevant area on each separate curve. For the two-sided alternative, include both equally extreme tails.

1. Calculate the common z statistic from the sample proportion 72/120, the null proportion 0.50, and the null standard deviation. Enter z in the organizer.

S07.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. For H_a: $p > 0.50$, use the first standard-normal curve in the organizer, label the observed z, shade the right tail, and calculate its area. Label this as the greater-than test.

3. Use the second curve in the organizer for H_a: $p < 0.50$. Mark the same observed z, shade the left tail, and calculate its area.

4. Use the third curve in the organizer for H_a: $p \neq 0.50$. Mark both $-|z|$ and $+|z|$, shade the two tails, and calculate their total area. State the decision at alpha = 0.05 for each of the three prespecified questions.

S07.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

5. Explain why it would be inappropriate to choose the smallest p-value after doing all three calculations.

Discuss your results

- Which part of the numerical calculation stays fixed across the three questions?
- What would a negative observed z do to the one-sided conclusions?

INDEPENDENT APPLICATION

1. An SRS of 200 of 10,000 customers finds 132 satisfied. Test H0: p = 0.60 versus Ha: p > 0.60 at alpha 0.05.

S07.02 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. An SRS of 80 of a club's 4,000 members finds that 42 favor a proposed schedule. Do the data provide convincing evidence that fewer than 60% of all club members favor the schedule? Use a significance level of 0.05 and justify your answer with an appropriate test.

3. For n=100 and p0=0.03, the observed success count is 15. Is the ordinary z test justified?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

An observed z is -1.80 for Ha: p > p0. Should the p-value be small or large? Explain.

S07.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 4, 5. **Optional practice.** 3, 6. **Prepare for the next meeting.** Read PR32 for S07.03 and answer both checks.

1. A company investigates whether the proportion of households with exactly three cell phones differs from 0.30. An SRS of 150 households from a population of 10,000 contains 43 with exactly three cell phones. At the 0.05 significance level, do the data provide convincing evidence that the population proportion differs from 0.30? Justify your answer with an appropriate test.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 9.5, Example 9.18, printed pp. 477-478 (PDF pp. 489-490). Original count, sample size, benchmark, and context retained; random sampling, population size, and alpha are specified.

2. An SRS of 200 of 12,000 adults contains 114 approving a plan. Test p=0.50 versus p not equal to 0.50 at alpha 0.05.

3. An SRS of 150 of a clinic's 8,000 clients finds that 84 report being satisfied with its appointment system. Do the data provide convincing evidence that fewer than 60% of all clinic clients are satisfied? Use alpha = 0.05. Conduct the appropriate test and interpret its p-value.

S07.02-HW | Continued

4. A student tests p0=0.50 with 57 successes in 100 observations but uses sqrt (0.57 times 0.43/100) in the denominator. Correct the computation and give the greater-than p-value.

5. Cumulative: Explain why an interval for 28/100 uses observed counts 28 and 72 while a test against p0=0.20 uses expected counts 20 and 80.

6. A software report rounds the p-value to 0.0000. The unrounded value is 0.00000301. Write a useful report and explain why practical importance still needs separate assessment.

S07.03 | Errors, power, and consequences

CED 3.8

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Apply one rule under two truths

Use the same rule as the worked case: 100 independent trials; reject H0: p=0.50 for 60 or more successes. Run one simulation with true p=0.50 and another with true p=0.65. Fallback illustrative outcomes for 1,000 studies: 28 rejections under 0.50; 875 rejections under 0.65.

Materials. Calculator or spreadsheet binomial random generator; two labeled tally columns.

Classify the outcomes before computing rates

Decision	True p = 0.50	True p = 0.65
Reject at 60 or more successes	Outcome name: _____	Outcome name: _____
Do not reject	Outcome name: _____	Outcome name: _____
Supplied example results	28 rejections / 1,000 studies	875 rejections / 1,000 studies
If using live simulation	Rejections: ____ / studies: ____	Rejections: ____ / studies: ____
Compute the relevant rates	Type I estimate: _____	Power: _____ Type II estimate: _____

Choose supplied or live results and identify that route. Each study contains 100 binary outcomes. Keep the two generating truths and their denominators separate.

1. In the decision-by-truth table, label all four outcomes as a correct decision, Type I error, or Type II error. Use H_0: $p = 0.50$ and H_a: $p > 0.50$.

2. Simulate at least 100 complete studies under $p = 0.50$ and at least 100 under $p = 0.65$, with 100 binary trials in each study. Count rejections separately for the two true proportions. If your teacher assigns the supplied counts instead, use 28/1,000 and 875/1,000 and identify them as supplied simulation results.

3. For the $p = 0.50$ studies, calculate the rejection proportion as an estimate of the Type I error probability. For the $p = 0.65$ studies, calculate the rejection proportion as estimated power and the nonrejection proportion as estimated Type II error probability. Enter these in the organizer and show each denominator.

S07.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Imagine changing the threshold from 60 to 58 successes. Without calculating, predict the effect on Type I error and power.

5. Write an interpretation of each estimated probability that explicitly states the assumed true p.

Discuss your results

- Why is it wrong to combine both truth conditions into one rejection percentage?
- Can a consequence analysis tell us what alpha must be without a value judgment?

INDEPENDENT APPLICATION

1. A test of p=0.10 against p>0.10 has power 0.82 when the true p=0.16. Interpret power and find beta at 0.16.

2. A safety process tests whether its failure rate exceeds 0.01. Describe both errors in context.

3. With n fixed, alpha drops from 0.05 to 0.01. Describe expected changes in Type I error, power, and Type II error at a fixed alternative.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A test of H_0: p = 0.50 against H_a: p > 0.50 has power 0.75 when the true proportion is 0.60. Must its power also be 0.75 when the true proportion is 0.51? Explain, assuming the sample size, significance level, and method stay fixed.

S07.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR33 for S07.04 and answer both checks.

1. In a textbook scenario, a treatment is claimed to have a cure proportion of 0.75. A researcher tests H_0: $p = 0.75$ against H_a: $p < 0.75$, where p is the population proportion of eligible patients cured under the treatment. Describe a Type I error and a Type II error in context. Give one possible consequence of each.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 9.2, Example 9.8, printed p. 466 (PDF p. 478). Source treatment benchmark and error comparison retained. Equality null, explicit population parameter, and careful nonrejection wording supplied.

2. At a specified true alternative, 2,000 simulated studies produce 1,640 rejections. Estimate power and beta, and explain which errors the remaining 360 represent.

3. A separate simulation under H0 produces 104 rejections in 2,000 studies. Estimate the Type I error probability. Is a nominal 0.05 rule contradicted by this count alone?

S07.03-HW | Continued

4. Compare two otherwise identical designs: A uses 100 observations and B uses 400. Predict the effect on standard error and power at the same nonzero departure.

5. Cumulative: For a one-sided test, a p-value is 0.03. Explain decisions at alpha 0.05 and 0.01, and explain why neither decision gives the probability of a Type II error for this particular study.

6. A researcher says, We failed to reject, so there is no meaningful effect; our study had only 30% power for the smallest meaningful effect. Critique the conclusion and propose a useful next step.

S07.04 | Test a difference in population proportions

CED 3.12, 3.13

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Pooling answers a null question

Independent SRSs find that 108 of 180 service-A users and 80 of 160 service-B users favor online renewal. Each service population contains at least 5,000 users. Let pA and pB be the corresponding population support proportions. Test H_0: pA = pB against H_a: pA ≠ pB.

Materials. Calculator; two-way success/failure table.

Construct the equal-proportions null model

Quantity	Group A	Group B
Given successes and n	108 out of 180	80 out of 160
Observed failures	_____	_____
Observed proportion	_____	_____
Pooled common proportion	(_____ + _____) / (_____ + _____)	_____
Null expected successes	_____	_____
Null expected failures	_____	_____
Two-sided test results	z = _____	p-value = _____

These are independent SRSs from populations of at least 5,000 each. Check the sampling fractions as well as the four null expected counts.

1. Compute both sample proportions and their ordered difference.

S07.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. In the pooled-proportion row, enter total supporters divided by total sampled users. Calculate that value and compare it with the unweighted average of the two sample proportions.

3. Complete the two rows of null expected counts in the organizer. Explain why those four counts, the stated sampling design, and each sample fraction justify the test.

4. Calculate the pooled SE, z, and two-sided p-value. Make the alpha 0.05 decision.

S07.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

5. Compute the unpooled interval SE from the same data. Explain why it is different without calling either formula a mistake.

Discuss your results

- What equality assumption licenses pooling?
- Would the same calculations justify inference from a pair of voluntary website polls?

INDEPENDENT APPLICATION

1. Independent SRSs from two service populations of 10,000 users each find that 150 of 250 service-A users and 110 of 220 service-B users favor electronic bills. Do the data provide convincing evidence that the population support proportions differ? Use alpha = 0.05 and justify your answer with an appropriate test.

S07.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. Why is the simple average of 0.80 from n=20 and 0.50 from n=100 not the pooled proportion? Find the pooled value.

3. A new training method and an old method were chosen by participants. A test finds a small p-value. What causal conclusion is justified?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Groups A and B have 60/100 and 54/120 successes. For a test of equality, show which counts you pool and explain why.

S07.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 4, 6. **Optional practice.** 3, 5. **Prepare for the next meeting.** Read PR34 for S07.05 and answer both checks.

1. Two types of valves are tested for cracking below 4,500 psi. Independent random samples from large production populations include 100 type-A valves, of which 15 crack, and 100 type-B valves, of which 6 crack. Assume each population contains at least 1,000 valves. Do the data provide convincing evidence that the population cracking proportions differ? Use alpha = 0.05. Justify your answer with an appropriate test.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 10.3, Try It 10.8, printed p. 524 (PDF p. 536). Counts, pressure threshold, and test question retained. Independent random samples, population sizes, and AP pooled expected-count checks are explicit.

2. For the valve data in question 1, explain why the two-proportion z test meets its large-count condition even though only 6 type-B valves cracked. Would the ordinary two-proportion z confidence interval meet its large-count condition? Justify your answer.

3. In a randomized experiment, 120 participants receive reminder A and 120 receive reminder B. Of these, 84 in A and 66 in B respond within 48 hours. Do the data provide convincing evidence that the reminders produce different response proportions? Use alpha = 0.01. Conduct an appropriate test and state the scope of your conclusion.

S07.04-HW | Continued

4. A student writes H0: p-hat 1=p-hat 2 and computes a pooled proportion by averaging observed proportions despite unequal n. Identify both errors.

5. Cumulative: A 95% interval for p1-p2 is (0.01,0.19). Explain what it says about zero and about a practically necessary difference of 0.15.

6. Two treatments are tried on each of 100 people, with binary outcomes for each. Explain why applying an independent two-proportion z test would be problematic and identify the design feature that matters.

S07.05 | Compare categorical distributions with chi-square

CED 3.14, 3.15

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Build the table the null predicts

Use the illustrative transit table: district A bus 40, bicycle 35, walking 25; district B bus 25, bicycle 35, walking 40. Each district sample is an independent SRS of 100 from at least 5,000 adults.

Materials. Calculator; blank observed and expected tables.

Expected-count workspace

Build the expected table and cell contributions

Given observed rows: A = (40, 35, 25); B = (25, 35, 40).

District	Bus	Bicycle	Walking
A expected	_____	_____	_____
B expected	_____	_____	_____

One cell: E = (row total × column total) / grand total = _____

Cell contributions: A bus ____ A bicycle ____ A walk ____

B bus ____ B bicycle ____ B walk ____

χ^2 = _____ df = _____ right-tail p-value = _____

All expected counts > 5? _____ Conclusion: _____

Expected counts, contributions, degrees of freedom, and the decision are intentionally blank.

1. Compute row totals, column totals, and pooled response proportions.

2. Complete the expected-count table using E = (row total)(column total)/(grand total). Show one calculation and verify that the expected row and column totals match the observed totals.

3. For each of the six cells, calculate $(O-E)^2/E$ in the organizer. Identify the cells with the largest contributions and use O−E to state the direction of each discrepancy.

S07.05 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Sum the contributions, identify df, and find the right-tail p-value.

5. Rewrite the conclusion twice: once as an appropriate overall statement and once as an overclaim that the test does not establish.

Discuss your results

- Why can an expected count be 32.5 when half a person cannot be observed?
- Why does a small p-value not prove that all three response proportions differ?

INDEPENDENT APPLICATION

1. Three independent populations are sampled and responses fall in four categories. State the homogeneity hypotheses in words and find df.

2. A cell has observed count 10, row total 60, column total 90, and grand total 300. Find E and its chi-square contribution. Does E meet the expected-count condition for this cell?

3. A homogeneity test returns p=0.19. A student says the population distributions are identical. Evaluate the conclusion at alpha 0.05.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A two-by-three table has all observed counts above 5 but one expected count is 3.8. Is the usual chi-square procedure justified?

S07.05-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 4, 5, 6. **Optional practice.** 3. **Prepare for the next meeting.** Read PR35 for S07.06 and answer both checks.

1. Independent random samples of 100 family households and 200 single-adult households record their primary vehicle type. In the order sport, sedan, hatchback, truck, van/SUV, the counts are family households: (5, 15, 35, 17, 28); single-adult households: (45, 65, 37, 46, 7). Assume each population is at least ten times its sample size. Do the data provide convincing evidence that the population vehicle-type distributions differ? Use alpha = 0.05. Identify the method, verify conditions, show the expected counts, and report the statistic, degrees of freedom, p-value, and conclusion. Technology may calculate the statistic and p-value.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 11.4, Try It 11.8, Table 11.20, printed p. 580 (PDF p. 592). Original counts and categories retained. Observational units are clarified as households with one primary vehicle, and AP conditions are explicit.

2. For the family-household sport-vehicle cell in question 1, calculate its contribution to χ^2. Does the observed count lie above or below the null expectation? Explain what the squared contribution alone does not tell you.

3. Independent SRSs of 60 adults from each of two districts, each with 5,000 adults, record preferred transit option. In the order bus, bicycle, walking, the counts are A: (30, 20, 10) and B: (20, 25, 15). Conduct a chi-square test for homogeneity at alpha = 0.05. Technology may calculate the statistic and p-value.

S07.05-HW | Continued

4. A randomized experiment assigns 30 participants to each of three treatments and classifies results as improved, unchanged, or worse. Identify the method and explain whether the treatment assignment itself requires a 10% population check.

5. Cumulative: Why is the chi-square p-value right-tailed even though some observed counts lie below expected counts and others above?

6. A report gives chi-square=9.8, df=4, p=0.044 and concludes every group differs from every other group. Correct the inference and name the additional evidence needed for causal language.

S07.06 | Investigate association with chi-square

CED 3.14, 3.15

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Identical numbers, different studies

Use the illustrative table A:(40,35,25), B:(25,35,40), with columns in the order bus, bicycle, walking. Scenario 1: take independent SRSs of 100 adults from each of two districts and record transit preference. Scenario 2: take an SRS of 200 adults from one district, classify each by work-schedule category A/B and transit preference, and obtain the same table. Each source population has at least 5,000 adults.

Materials. Scenario cards; calculator; blank hypothesis statements.

Let the study design determine the question

Study design	Method and null hypothesis	Scope of a supported conclusion
Two independent SRSs: 100 adults from each district; one transit preference response.	_____	_____
One SRS: 200 adults from one district; work-schedule category and transit preference.	_____	_____
Random assignment to two travel-information messages; one response category per person.	_____	_____

The observed table can be identical across designs. Identify the sampling or assignment mechanism before naming the test or claiming causation.

1. Complete the first two rows of the design organizer. For each scenario, name the chi-square test and write its null and alternative hypotheses using the populations and variables in that scenario.

S07.06 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Explain the randomization and 10% checks appropriate to each design.

3. Compute the shared expected counts, statistic, df, and p-value.

4. For each scenario, write a conclusion at alpha = 0.05 in the organizer's scope column. Name the population or populations and the variables. Explain why neither observational design supports a causal conclusion.

S07.06 | Classwork, continued

GUIDED APPLICATION, CONTINUED

5. For the third organizer row, suppose participants are randomly assigned to one of two travel-information messages and report one transit preference. Name the test, state the hypotheses, and explain how random assignment changes the causal conclusion and how recruitment limits generalization.

Discuss your results

- Why cannot the numbers alone tell us which hypothesis wording is appropriate?
- Which conclusions require random assignment rather than only random sampling?

INDEPENDENT APPLICATION

1. An SRS of 100 adults from a district of 10,000 classifies bike ownership (yes/no) and cycling to work (yes/no). Counts are owners: (30,20), nonowners: (15,35). Test independence at alpha 0.05.

S07.06 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. Separate random samples from four regions classify each person's preferred one of three options. Which chi-square test and what df?

3. A sample of 300 students contains 50 music students and 97 honor-roll students. (a) Assuming the two characteristics are independent, calculate the expected number who are both music students and on the honor roll. (b) Can these marginal totals alone establish an association? Explain.

Source: OpenStax, Introductory Statistics 2e, supplied 2026 printing, section 11.3, Try It 11.5, printed p. 575 (PDF p. 587). Original counts and expected-count task retained. A second part distinguishes expected counts from evidence of association.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

An investigator takes one random sample and records two categorical variables. A colleague calls it homogeneity because the table has two rows. Correct the reasoning.

S07.06-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR36 for C4 and answer both checks.

1. An SRS of 200 of 10,000 adults records remote-work status and preferred news medium. Counts are remote:(40,30,30), not remote:(20,35,45) for print/audio/video. Test independence at alpha 0.05.

2. For the preceding table, calculate the conditional print preference in each row and identify a descriptive difference. Does this alone replace the inference test?

3. An independence test uses an SRS of 300 people from a population of 2,000. Every expected count exceeds 20. Identify the remaining failed condition.

S07.06-HW | Continued

4. Choose a method: (a) test one town's yes proportion against 0.40; (b) test whether program A has a higher yes proportion than program B in a randomized experiment with independent treatment groups; (c) compare three regions' distributions across four responses; (d) test association between two categorical variables in one SRS.

5. Cumulative: A chi-square test with p=0.008 rejects at alpha 0.01. State a possible Type I error in the remote-work/news-medium context.

6. A student enters row percentages as if they were observed counts and obtains a p-value. Explain why this can change the evidence, even if the graph looks identical.

Unit 7 | Cumulative synthesis

1. An SRS of club members is used to test whether more than half favor a new schedule. The test of H_0: $p = 0.50$ against H_a: $p > 0.50$ gives p-value 0.014, where p is the population support proportion. Interpret this p-value in context.

2. An SRS of 120 of a club's 6,000 members finds that 72 favor a proposed schedule. At the 0.05 significance level, do the data provide convincing evidence that a majority of all club members favor the schedule? Justify your answer with an appropriate test.

3. At a specified alternative, power is 0.91. Find beta and interpret it.

4. Explain when a proportion standard error is pooled and when it is not.

5. A two-by-four table is used for homogeneity. Find df, identify the tail, and state the expected-count requirement used in this course.

6. Explain the difference between homogeneity and independence and one limitation shared by observational versions of both.

Unit 08 | Confidence Intervals for Means

Quantitative responses lead to questions about population means and mean differences. This unit develops t-based confidence intervals from the role of estimated variability, then separates one-sample, paired, and independent-group designs. Students practice checking conditions with evidence, interpreting confidence in context, and judging claims by the entire interval. Investigations show why sample size and pairing affect precision. The local worked examples and investigations use illustrative datasets; credited source examples retain their stated provenance. Technology performs arithmetic, while students remain responsible for the parameter, design, assumptions, and scope of the conclusion. The unit closes by connecting inference choices to the question that motivated data collection.

CED topics: 4.2, 4.3, 4.7, 4.8

IN THIS UNIT

Lesson	Focus
S08.01	Estimating a population mean
S08.02	Confidence, precision, and defensible claims
S08.03	Paired observations: analyze the differences
S08.04	Estimating a difference between independent means

Match the times before choosing a procedure

Walker	Route A (min)	Route B (min)
K	16	19
L	21	20

Illustrative times. Both measurements on a row belong to the same walker.

LOCAL QUESTION

How could you estimate mean travel time along a fixed trail segment? Decide whether comparing two routes would call for paired or independent observations.

S08.01 | Estimating a population mean

CED 4.2

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

How much does estimating variability cost?

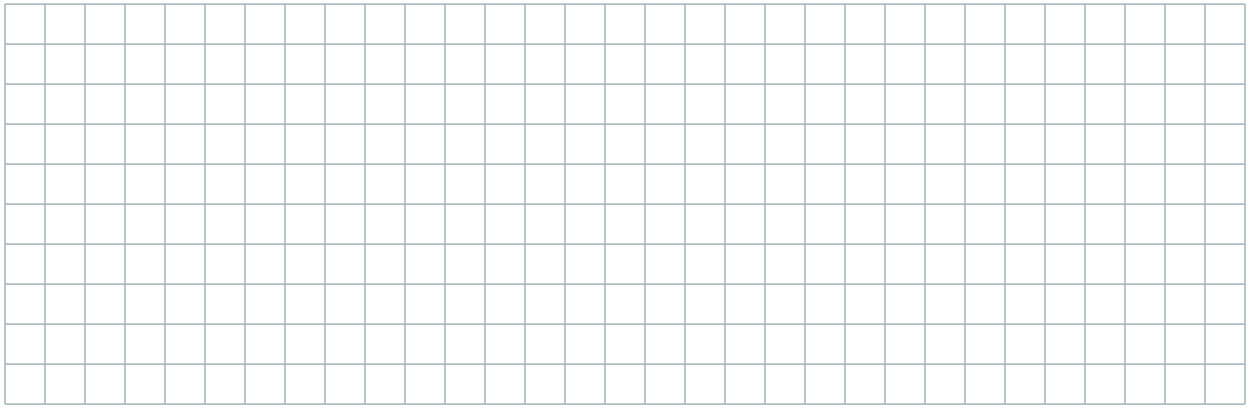
Use these illustrative summary samples from the same quantitative population: A has n = 9, mean = 50, s = 6; B has n = 25, mean = 50, s = 6; C has n = 100, mean = 50, s = 6. Assume independent random samples, population at least 10,000, and sample plots without strong skewness or outliers.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Predict the order of interval widths before calculating.

2. Find SE and the 95% t critical value for A, B, and C.

3. Construct all three 95% confidence intervals. Draw their horizontal segments on one common numerical scale and label them A,B,C.



4. For A, compare the t margin of error with the margin obtained using 1.96. Explain the difference.

Discuss your results

• Why does sample A have a larger t multiplier than sample C at the same confidence level?

• Why is it important that these planning samples were constructed with the same mean and sample standard deviation?

S08.01 | Practice and exit

INDEPENDENT APPLICATION

Build the package-mass interval

Quantity	Given value or student work
Sample size / population	n = 36; N = 2,000; SRS
Sample mean / sample SD	510 g / 18 g
Confidence level	95%
df; standard error	_____
Critical value; margin of error	_____
Population-mean interval	_____

Use the first independent-practice dataset. Complete the calculation and explain each quantity.

1. An SRS of 36 packages from 2,000 has mean mass 510 g and s = 18 g. Find a 95% t-interval and interpret it.

2. A convenience sample of 60 customers gives a very narrow interval for mean wait. Explain why the narrow interval does not justify generalizing to all customers.

3. For a sample of 16 with s = 8 minutes, identify s, SE, and df and explain each.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Why is a t-distribution appropriate when n = 80 and the population standard deviation is unknown?

S08.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 5. **Optional practice.** 6. **Prepare for the next meeting.** Read PR38 for S08.02 and answer both checks.

1. The OpenStax sleep example gives 12 observations, in hours: 8.2, 9.1, 7.7, 8.6, 6.9, 11.2, 10.1, 9.9, 8.9, 9.2, 7.5, 10.5. Assume an independent random sample from a normal population of at least 120 subjects. Construct and interpret a 95% confidence interval for the population mean nightly sleep duration.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 8.2, Try It 8.8, p.419 (PDF 431); sampling assumptions clarified.

2. Using the same sleep data from question 1, construct a 99% confidence interval for the population mean nightly sleep duration. Explain why the interval is wider than the 95% interval.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 8.2, Try It 8.8, p.419 (PDF 431); data unchanged.

3. A sample of 12 completion times has one extremely high outlier. Explain why checking $n \le 10\% N$ is insufficient.

S08.01-HW | Continued

4. If s = 15 cm and n = 100, calculate SE. A classmate says the interval will contain 95% of measured lengths. Correct the statement.

5. Cumulative: From a school population of 500, describe an SRS of 20 students that another person could reproduce.

6. An interval is (41,47) minutes, centered on a sample mean. Recover its point estimate, margin of error, and width; explain what is unknown.

S08.02 | Confidence, precision, and defensible claims

CED 4.3

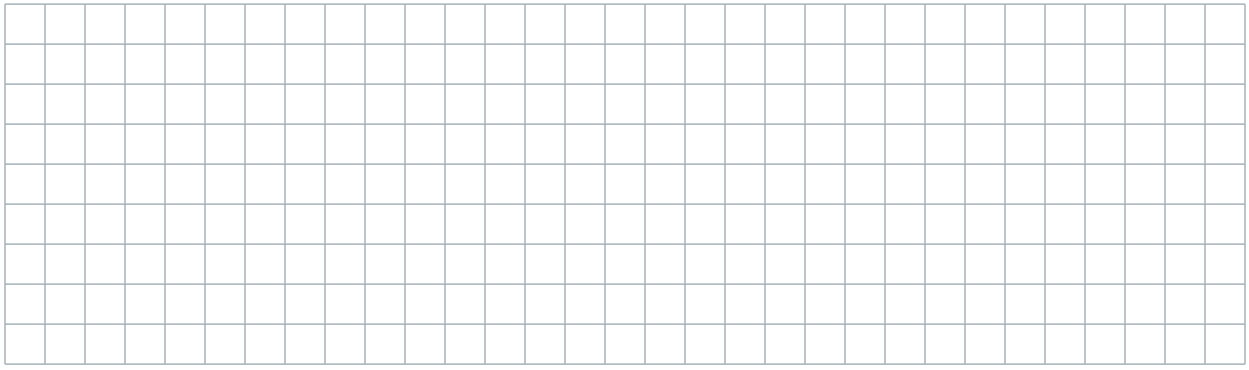
GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Which intervals capture the mean?

For a deliberately illustrative coverage exercise, the known population mean is 50. Ten fictional 95% intervals are (47,53),(48,54),(44,49),(49,55),(46,52),(48,52),(45,51),(50.5,56.5),(47,51),(49,53). These are supplied teaching intervals, not a simulated estimate of the true coverage rate.

Materials. Graphing calculator or statistical software; pencil and paper.

1. On a common horizontal scale, draw a separate horizontal segment for each of the ten supplied intervals. Label the segments 1-10 and draw a vertical reference line at the known mean 50.



2. Count how many capture the population mean.

3. Explain why your observed fraction need not equal 95%.

4. Predict the effect of using 99% intervals from the same underlying samples.

Discuss your results

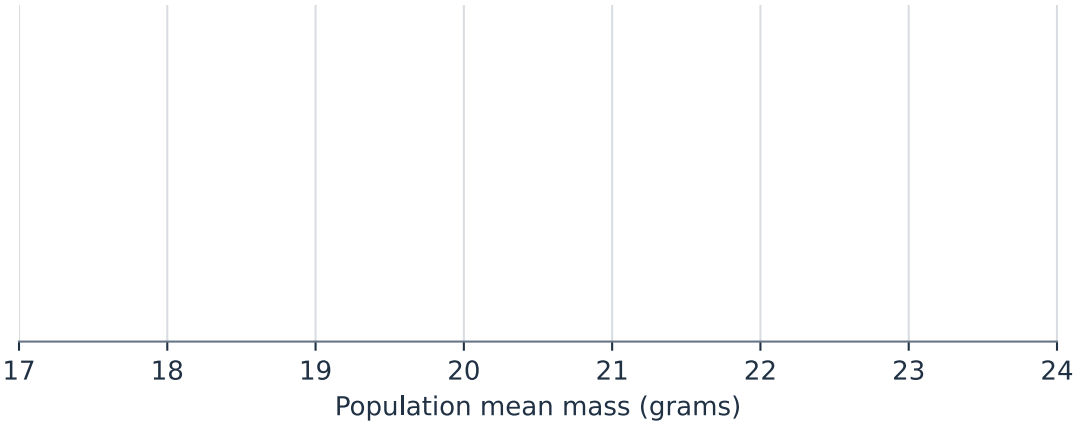
• What stays fixed and what changes from interval to interval in a repeated-sampling interpretation?

• Can this deliberately selected set of ten intervals be used to estimate the actual coverage rate of the 95% method? Explain.

S08.02 | Practice and exit

INDEPENDENT APPLICATION

Plot an interval and evaluate two claims



Given: a 95% interval is (18.2, 21.6) grams. Draw it, then mark candidate means 20 and 23. Explain what the picture supports.

1. A 95% interval for a population mean is (18.2,21.6) grams. Is 20 compatible? Is 23 compatible?

2. A 99% interval is wider than a 90% interval from the same data. Explain without saying merely "because confidence is higher."

3. For approximately the same variability and confidence, how must n change to reduce margin of error from 4 to 2?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

A 95% interval contains 0.4 but excludes 0. A student says the parameter is definitely 0.4. Explain the error.

S08.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 5, 6. **Optional practice.** 4. **Prepare for the next meeting.** Read PR39 for S08.03 and answer both checks.

1. An SRS of 64 trail users from 4,000 has a mean visit time of 72 minutes and s = 16 minutes. Construct a 95% interval and assess a claim that the population mean is 67 minutes.

2. Use the same sample to construct a 99% interval. Does the wider interval still exclude 67 minutes?

3. Write separate correct statements interpreting a 95% interval (62,70) minutes and the 95% confidence level for a defined population of comparable trail visits.

S08.02-HW | Continued

4. A study doubles sample size from 50 to 100, keeping s and confidence approximately fixed. Does margin of error halve? Show the approximate factor.

5. Cumulative: A random sample includes only users who bought tickets online. The target is all users. Identify the design issue and explain why a narrow interval does not resolve it.

6. For the OpenStax sensory-score data, a 95% interval for the population mean is (7.30,9.15) score units. An analyst claims that 95% of the subjects have scores in that interval. Explain why the claim is incorrect and give a correct interpretation.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 8.2, Example 8.8, pp.418-419 (PDF 430-431); interpretation question adapted.

S08.03 | Paired observations: analyze the differences

CED 4.2, 4.3

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Hands-on option. Use this investigation or "Does the launch surface change a frog's jump?," then continue to Independent application.

Does the pairing matter?

Use illustrative paired measurements A=(10,20,30,40,50) and B=(8,19,27,38,48), in seconds. These five pairs are for comparing variability only, not for a formal population claim.

Materials. Graphing calculator or statistical software; pencil and paper.

Preserve the pairs before calculating

Pair	A (seconds)	B (seconds)	A − B (seconds)
1	10	8	_____
2	20	19	_____
3	30	27	_____
4	40	38	_____
5	50	48	_____

Use the investigation's given pairs. Create the difference column without rearranging either measurement column.

1. Compute A−B for the original pairs and find the sample standard deviation of differences.

S08.03 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. On the response lines, make a second difference list by reversing only B to(48,38,27,19,8), keeping A fixed. Calculate the sample standard deviation of this new difference list. Keep the original table unchanged.

3. Compare the two mean differences. Why do they agree?

4. Explain why the reversed pairing is not a legitimate alternative analysis if the original pairs are meaningful.

Discuss your results

• Why does reversing the B list leave the mean difference unchanged but change its standard deviation?

• What feature of a real study determines whether the original row pairing is valid?

S08.03 | Optional lab | Does the launch surface change a frog's jump?

Optional guided application | 20 minutes

Measure the same launcher and frog on both surfaces and describe the mean change in jump distance for the observed class pairs.

Materials. Plastic jumping frogs, two launch surfaces, tape, meterstick or tape measure, and a coin.

COLLECT YOUR DATA

1. Label the two surfaces A and B and record their names. Each pair consists of one person and one frog. Flip a coin for that pair's order. Use A first for heads or B first for tails.

2. Use the same person, frog, and launch technique on both surfaces. Place the frog with its front toes on the start line. Measure forward from that line to its front toes at the first landing, before it slides, to the nearest centimeter. Before collecting data, agree how to record a sideways jump or failed launch; apply the rule to both surfaces.

3. Compute A minus B for your pair. Enter your pair and three other pairs in the table as a recording check. The teacher will display the complete class list. Use every distinct class pair once when calculating the class mean difference, even if only four pairs appear on your sheet.

Launcher-frog ID	Order	A (cm)	B (cm)	A − B (cm)
Our pair				
Class pair				
Class pair				
Class pair				

ANALYZE YOUR RESULTS

1. Why would independently sorting the two columns before subtracting change the data being analyzed?

2. In one sentence, report the two surface names, number of class pairs, direction and size of the observed mean difference in centimeters, and who or what your comparison describes.

Continue to this lesson's Independent application.

S08.03 | Practice and exit

INDEPENDENT APPLICATION

1. Twenty students each use two interfaces in randomized order. What are n and df for the paired interval?

2. The 95% interval for old–new time is (1.1,3.9) seconds. Interpret it and give the interval for new–old.

3. A class is tested before and after a month of tutoring, with no control group. Why does a positive paired difference not establish that tutoring caused improvement?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Two independent groups each have 18 people. May their observations be paired by rank to use a paired t-interval?

S08.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR40 for S08.04 and answer both checks.

1. Using the seven shot-put differences from the source example (2,12,7,−1,2,0,4 feet), construct and interpret a 90% confidence interval for the population mean dominant-minus-weaker distance. Use the same sampling and normality assumptions.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 10.4, Example 10.13, pp.531-532 (PDF 543-544);90% interval extension.

2. For the same seven students, define each difference as weaker-hand distance minus dominant-hand distance. State the new mean difference, sample standard deviation, and 90% confidence interval. Explain which quantities change.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 10.4, Example 10.13, pp.531-532 (PDF 543-544).

3. A student checks the before and after plots but never plots the ten differences. Identify the missing evidence.

S08.03-HW | Continued

4. A paired 95% interval is $(-1.4, 2.6)$ points. A researcher reports "the treatments are equivalent." Explain what the interval does and does not support.

5. Cumulative: Random selection and random treatment order have different purposes. State each for a crossover device study.

6. A lab has only the two sample means and sample standard deviations for paired data. Can it reconstruct the paired interval uniquely? Explain.

S08.04 | Estimating a difference between independent means

CED 4.7, 4.8

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Hands-on option. Use this investigation or "Which package has more visible chocolate patches per cookie?," then continue to Independent application.

Which sample needs more observations?

Use illustrative planning summaries: sample A has n=25 and s=10 seconds; sample B has n=100 and s=10 seconds. Treat these as estimates for planning; no population claim is being tested.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Calculate each group's contribution to SE^2 and the combined SE.

2. Recalculate SE if 50 additional observations go to A.

3. Recalculate SE if the same 50 additional observations go to B.

4. Explain which allocation reduces uncertainty more and why.

Discuss your results

- Which variance contribution limits precision most in the original allocation?
- Would the same allocation necessarily be best if an observation in A cost much more than one in B? Explain.

S08.04 | Optional lab | Which package has more visible chocolate patches per cookie?

Optional guided application | 20 minutes

Compare mean counts of visible chocolate patches for the cookies sampled from two packages and identify the unit needed for a broader brand comparison.

Materials. One package each of comparable original-style Chips Ahoy! and WinCo chocolate chip cookies, two trays, cookie ID labels, and a random-number generator.

COLLECT YOUR DATA

1. Label cookies within each package before looking at their tops. For each brand, draw one list of distinct IDs at random without replacement and give one selected cookie to each group. Keep the package and cookie IDs with each observation.

2. Put the flat underside down. Count each separate visible patch of chocolate on the top face once, including a partial chip. Count touching chocolate as one patch. Exclude the sides and underside. Two students count the same cookie and agree on one recorded count.

3. Enter your counts and a second group's counts below. Pool each cookie only once. Compute the mean for each package's sampled cookies and the difference, Chips Ahoy! minus WinCo.

Brand and package	Cookie ID	Patches on top
Chips Ahoy! package A		
Chips Ahoy! package A		
WinCo package B		
WinCo package B		

ANALYZE YOUR RESULTS

1. If every group counted two more cookies from the same packages, what source of variation would still be missing?

2. In one sentence, compare the two sampled groups using the difference in mean visible chocolate patches per cookie, and state how far the evidence reaches.

Continue to this lesson's Independent application.

S08.04 | Practice and exit

INDEPENDENT APPLICATION

Two groups, two uncertainty contributions

Quantity	Group 1	Group 2
n	25	40
Sample mean	12	10
Sample SD	3	4
Variance contribution s^2/n	_____	_____
SE for the difference	_____	

Use the first independent-practice comparison. Keep each group's sample statistics together.

1. Independent random samples have $n_1 = 25$ and $n_2 = 40$, means 12 and 10, and sample SDs 3 and 4. Show each variance contribution and find the unpooled SE for the difference.

2. Let μ_A and μ_B be the population mean times, in seconds, to complete the same task using interfaces A and B. A 95% confidence interval for $\mu_A - \mu_B$ is $(-6, -2)$ seconds. Interpret the interval in context.

S08.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

3. Each of 20 students completes both tests. A classmate uses an independent two-sample interval. Explain the design error.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Why do the two variance terms add in the standard error for a difference of independent means?

S08.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 3, 4, 5, 6. **Optional practice.** 2. **Prepare for the next meeting.** Read PR41 for S09.01 and answer both checks.

1. For the two battery brands in the source example, construct a 90% confidence interval for $\mu_A - \mu_B$ using the given independent-sample summaries and assumptions. Interpret the interval in context.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 10.1, Try It 10.1, p.514 (PDF 526); battery context and 90% interval task adapted.

2. For your 90% battery-life interval, report the point estimate and margin of error. Does the interval give convincing evidence that the brands have different population mean lifetimes? Explain.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 10.1, Try It 10.1, p.514 (PDF 526).

3. A: n=8 with no outliers; B: n=60. No graph is provided for either group. Can sample size alone justify the two-sample procedure?

S08.04-HW | Continued

4. Two separate 95% mean intervals overlap. Does this prove that a 95% interval for their difference contains 0? Explain.

5. Cumulative: Suppose A and B are randomized treatments assigned to volunteer participants. Explain the strongest scope of a supported mean difference.

6. The interval for $\mu_A - \mu_B$ is $(1.5, 4.7)$. Give the interval for $\mu_B - \mu_A$ and explain what remains unchanged.

Unit 8 | Cumulative synthesis

1. Explain the roles of s and $s/\sqrt{n}$.

2. An SRS of 25 has mean 30 and s = 10. Show the standard error and find a 95% mean interval when the plot is suitable.

3. Twenty subjects each receive both conditions. What is the analysis unit?

4. An interval for $\mu_A - \mu_B$ is $(-4,1)$. What claim is unsupported?

5. How does quadrupling n affect the margin of error approximately?

6. Why can a narrow interval from convenience data be misleading?

Unit 09 | Hypothesis Tests for Means

This unit uses quantitative data to evaluate claims about population means. Students apply one-sample, paired, and independent two-sample t tests, with emphasis on selecting the method from the study design. Every test begins with a defined parameter and a question-based alternative, then checks conditions before interpreting a p-value. The final lesson compares inference families across the course and connects matching t tests with confidence intervals. Students distinguish statistical evidence from practical importance and separate mean change from causal explanation. Original illustrative investigations develop these ideas before students complete independent contextual responses and mixed homework.

CED topics: 4.4, 4.5, 4.9, 4.10

IN THIS UNIT

Lesson	Focus
S09.01	Testing a claim about one population mean
S09.02	Testing a population mean difference from paired data
S09.03	Testing a difference between independent means
S09.04	Choosing inference and linking tests with intervals

A paired record for storage loss

Pear	Before (g)	After (g)	Before − after (g)
A	174	166	8
B	190	187	3

Illustrative measurements. A difference of 8 g is the mass loss for one pear.

LOCAL QUESTION

Would measuring each pear before and after storage create paired data? Define the difference and the population before choosing a test.

S09.01 | Testing a claim about one population mean

CED 4.4, 4.5

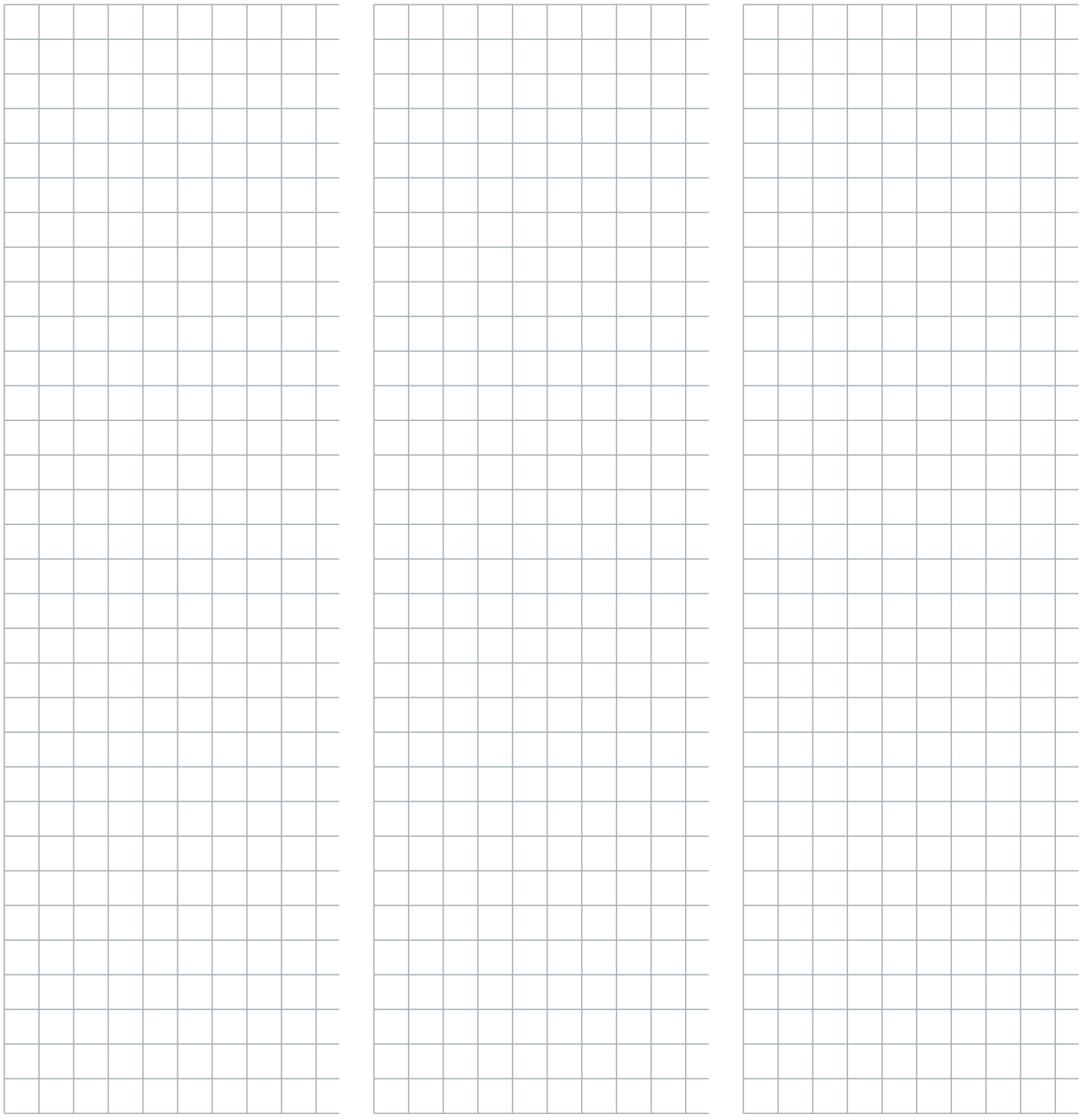
GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

One statistic, three investigative questions

Use the illustrative packet result t=2.0000 with df=24. Three questions were specified independently: Is the mean above 100? Below 100? Different from 100?

Materials. Graphing calculator or statistical software; pencil and paper.

1. Sketch the appropriate tail or tails for each alternative.



S09.01 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Find all three p-values with the same t distribution.

3. Make the three decisions at significance 0.05.

4. Explain why choosing the smallest p-value after seeing results is not a valid way to choose the question.

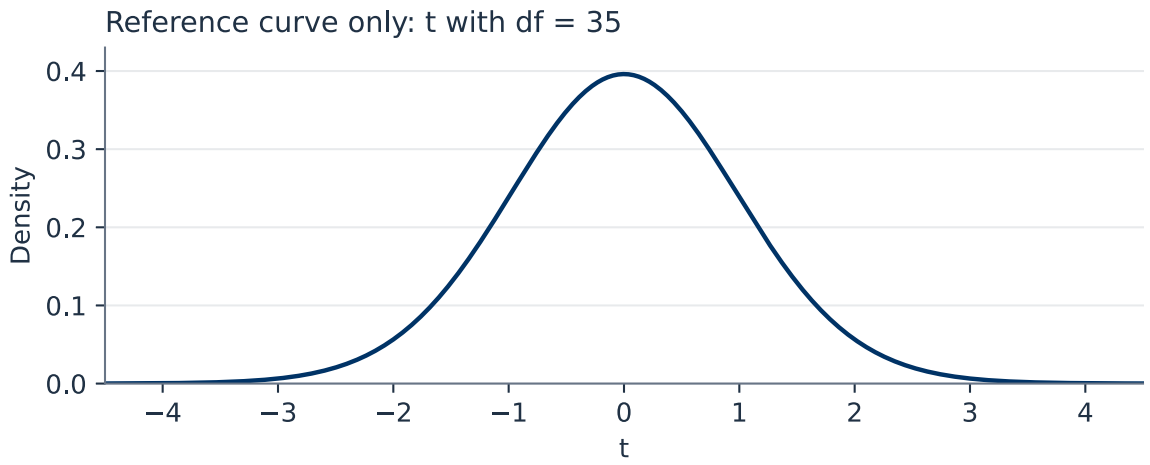
Discuss your results

- Why is the lower-tail p-value close to 1 when the observed statistic is positive 2?
- Why can the upper-tail and two-sided tests reach different decisions at 0.05?

S09.01 | Practice and exit

INDEPENDENT APPLICATION

Calculate, locate, and shade



Given practice data: n = 36, mean = 12.4, s = 3; H_0: $\mu = 11$, H_a: $\mu > 11$. Calculate t, mark it, and shade the p-value region. No observed statistic or tail is supplied.

1. A random sample of 36 comparable print jobs has mean completion time 12.4 minutes and s=3 minutes. Assume the t-test conditions are met. Test H_0:μ=11 minutes against H_a:μ>11 minutes at α=0.05. On the supplied t curve, mark the observed statistic and shade the p-value region. State the contextual conclusion.

2. A p-value is 0.08 at significance 0.05. State the decision without claiming equality.

S09.01 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

3. A report states H_0: $\bar{x}=100$. Correct it and explain.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

With a two-sided alternative and $t=-2.4$, which probability is the p-value?

S09.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 5. **Optional practice.** 6. **Prepare for the next meeting.** Read PR42 for S09.02 and answer both checks.

1. Using the ten scores and assumptions in the OpenStax first-test example, test H_0:μ=65 points against H_a:μ>65 points at α=0.01. Show the test statistic, p-value, and contextual conclusion.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 9.5, Example 9.16, pp.475-476 (PDF 487-488); significance level changed to 0.01.

2. For the same test-score sample, suppose the alternative had instead been specified as H_a:$\mu\neq$65 points. Calculate the two-sided p-value and state the decision at α=0.05. Explain why it differs from the one-sided decision at 0.05 in the reference.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 9.5, Example 9.16, pp.475-476 (PDF 487-488).

3. A p-value of 0.03 is reported for a greater-than mean alternative. Write its interpretation using a null mean of 50 grams.

S09.01-HW | Continued

4. An automated analysis gives p=0.0004 from an online volunteer sample. Is a claim about all residents justified?

5. Cumulative: An ordinary 95% t-interval for μ is (97,103). What would the matching two-sided 5% test of μ=100 conclude?

6. A mean difference of 0.02 seconds is statistically significant in a large study. Name two additional facts needed for a useful conclusion.

S09.02 | Testing a population mean difference from paired data

CED 4.4, 4.5

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Reverse the definition without changing the question

Use the illustrative interface summaries n=12, mean old−new=1.5 seconds, s_d=2 seconds. The investigative question remains whether the new interface reduces completion time.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Write hypotheses using old−new. Compute t and the p-value.

S09.02 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. Now define new−old. Rewrite both hypotheses and compute the new t.

3. Show that the appropriate p-value is unchanged.

4. Explain why changing only the sign of the statistic while leaving the alternative unchanged would test a different question.

Discuss your results

• When the difference order reverses, which elements of the test must change together?

• Would reversing the subtraction order change the conclusion about which interface is faster? Explain.

S09.02 | Practice and exit

INDEPENDENT APPLICATION

One sample of differences

Given or required item	Value / student work
Number of pairs	10
Mean difference; sample SD	−2 seconds; 3 seconds
Null / alternative	_____
Degrees of freedom	_____
SE; observed t	_____
p-value; contextual conclusion	_____

Use the first independent-practice problem: test whether the population mean difference is negative. State hypotheses, then analyze the difference variable.

1. For 10 randomly selected paired device tests, let d=new-minus-old completion time in seconds. The mean difference is −2 seconds and s_d=3 seconds. Assume independence between pairs, the 10% condition, and an approximately normal difference population. Use the organizer to test $H_0: \mu_d = 0$ against $H_a: \mu_d < 0$ at $\alpha = 0.05$. State your conclusion.

2. A study of before/after resting time reports p=0.01. No control or randomization was used. What conclusion is too strong?

S09.02 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

3. A paired experiment has 18 pairs. Why is df 17 rather than 35?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

If the paired difference plot has one extreme outlier and n=8, what should the condition check say?

S09.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR43 for S09.03 and answer both checks.

1. For the eight subjects in the OpenStax source example, redefine d as before-minus-after sensory score. State hypotheses to test whether scores decrease after the procedure, calculate t and the appropriate p-value, and conclude at $\alpha=0.05$.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 10.4, Example 10.11, pp.528-529 (PDF 540-541); subtraction order reversed.

2. For the same eight subjects, suppose the question had asked whether the population mean score changed in either direction. State the alternative and calculate the two-sided p-value. Compare it with the one-sided p-value.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 10.4, Example 10.11, pp.528-529 (PDF 540-541).

3. Write a p-value interpretation for a paired test of mean before–after heart rate 0 with $p=0.04$ and an alternative above 0.

S09.02-HW | Continued

4. Two groups of 10 devices are independently selected. Their rows are sorted and subtracted. Explain why a paired t test is inappropriate.

5. Cumulative: An interval for mean old–new time is (0.2,1.4) seconds. Give the reversed interval and explain the practical comparison.

6. A crossover trial always gives the old method first. Identify a possible confound and propose an improvement.

S09.03 | Testing a difference between independent means

CED 4.9, 4.10

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Same difference, different evidence

Use three illustrative independent-group studies. All have group means 54 and 50. Study I: $n_1=n_2=16$, $s_1=s_2=8$. Study II: $n_1=n_2=64$, $s_1=s_2=8$. Study III: $n_1=n_2=16$, $s_1=s_2=4$. Assume random independent samples from large normal populations.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Predict which study gives the largest magnitude t statistic.

2. Calculate SE, t, and two-sided p-value for each.

3. Explain separately how sample size and variation change the strength of evidence.

4. Explain why identical mean differences do not imply identical practical contexts.

Discuss your results

- Why do studies II and III have equal test statistics but different p-values?
- What information beyond a p-value is needed to judge whether a four-unit mean difference matters?

S09.03 | Practice and exit

INDEPENDENT APPLICATION

Keep independent-group summaries aligned

Quantity	Group 1	Group 2
n	36	64
Mean (seconds)	21	18
Sample SD (seconds)	3	8
H_0 / H_a (two-sided)	_____	
SE; df; t; p-value	_____	

Use the first independent-practice test. Keep the two groups' sample summaries separate, then write one set of hypotheses and one analysis for the difference in population means.

09

1. Independent random samples of two types of print job give $n_1=36, n_2=64$, mean times 21 and 18 seconds, and sample SDs 3 and 8 seconds. Assume the two-sample t conditions are met. Test $H_0: \mu_1-\mu_2=0$ against $H_a: \mu_1-\mu_2\neq 0$ at $\alpha=0.05$. Use the organizer to record the hypotheses, SE, Welch df, statistic, p-value, and conclusion.

2. A group of 8 and a group of 100 are compared. A student writes "$n=108\geq 30$." Explain why the check is incomplete.

3. A randomized experiment reports $p=0.09$ at 0.05. What is the appropriate conclusion?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

Why is an unpooled two-sample t test not the same as the pooled two-proportion z test?

S09.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 5, 6. **Optional practice.** 4. **Prepare for the next meeting.** Read PR44 for S09.04 and answer both checks.

1. Use the College A and College B summaries in the OpenStax source example. Conduct the one-sided test for $\mu_A > \mu_B$ at $\alpha = 0.05$, using the same sampling and normality assumptions. Show the hypotheses, standard error, test statistic, p-value, and conclusion.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 10.1, Example 10.2, pp.514-516 (PDF 526-528); significance level changed.

2. Interpret the p-value from the college comparison in the context of the study. Include the null assumption and what would count as a result at least as extreme.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 10.1, Example 10.2, pp.514-516 (PDF 526-528).

3. Two independent methods have 95% interval for $\mu_A - \mu_B = (-0.8, 3.2)$. Predict the matching two-sided 0.05 test decision.

S09.03-HW | Continued

4. A report says a significant mean comparison proves every A observation exceeds every B observation. Correct it.

5. Cumulative: Two measurements are taken on each of 30 trees. Explain why the independent two-sample formula may waste or misrepresent information.

6. Design a study to compare two study-room layouts for completion time. State randomization, parameter, and one limit.

S09.04 | Choosing inference and linking tests with intervals

CED 4.2, 4.3, 4.4, 4.5, 4.7, 4.8, 4.9, 4.10, 1.1, 1.10

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Method sorting with reasons

Classify these original study plans: A estimates the proportion of students biking; B tests mean bike-trip duration against 15 minutes; C compares duration for the same riders on two routes; D compares mean duration in two independent towns; E examines association between commute mode and grade from one student sample.

Materials. Graphing calculator or statistical software; pencil and paper.

Choose the method from the design

Study goal and design	Procedure family
A: estimate proportion biking	_____
B: test one mean duration against 15 min	_____
C: same riders try two routes	_____
D: independent riders from two towns	_____
E: commute mode and grade in one sample	_____

Use the method-sorting investigation. Fill the final column before calculating anything.

1. Identify the response type and observational unit in each plan.

S09.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

2. In the organizer, identify a procedure family for each plan. On the response lines, explain the response type, pairing or independence, and estimation or testing goal that justify each choice.

3. Write one population parameter or categorical hypothesis for each.

S09.04 | Classwork, continued

GUIDED APPLICATION, CONTINUED

4. Revise one vague plan into a precise investigative question, including population, variable, and purpose.

Discuss your results

• Which feature distinguishes plan C from plan D even though both compare two routes or groups?

• What additional design information would you need before carrying out any selected inference procedure?

INDEPENDENT APPLICATION

1. A 92% two-sided t-interval is (1.2,5.6). What significance level matches a two-sided test of 0, and what is the decision?

S09.04 | Practice and exit

INDEPENDENT APPLICATION, CONTINUED

2. A one-sided 5% t test rejects. Must a two-sided 95% interval exclude the null?

3. A student asks “Is the school better?” Revise this into a measurable statistical question.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

What three questions should you ask before choosing an inference procedure?

S09.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR45 for P02 and answer both checks.

1. Choose a method and define the parameter: An SRS of 100 residents records whether each supports a policy; the goal is an estimate.

2. Choose a method: An SRS of 40 bottles records volume; the question is whether mean volume differs from 500 mL.

3. Choose a method: Independent random samples from three schools are classified into four commute modes; the question compares the distributions.

S09.04-HW | Continued

4. The same jobs are timed on printers A and B. A 95% paired t-interval for the mean A-minus-B printing time is (−3, −0.5) seconds. Give the matching two-sided test decision at $\alpha = 0.05$ and interpret the direction.

5. Cumulative: An observational association is statistically significant. Explain why an unmeasured confound remains relevant.

6. For each source study, identify an appropriate mean-test procedure and explain the design feature that determines your choice: (a) ten students' first-test scores compared with 65 points; (b) independent samples of graduates from two colleges; (c) before and after scores measured on the same eight subjects. Do not calculate.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Sections 9.5/10.1/10.4, Examples 9.16/10.2/10.11; design comparisons adapted.

Unit 9 | Cumulative synthesis

1. State hypotheses for testing whether population mean time is below 20 minutes.

2. For n = 25, sample mean 104, and s = 10, test $\mu = 100$ against a two-sided alternative. Show the standard error.

3. Why does a paired test use differences?

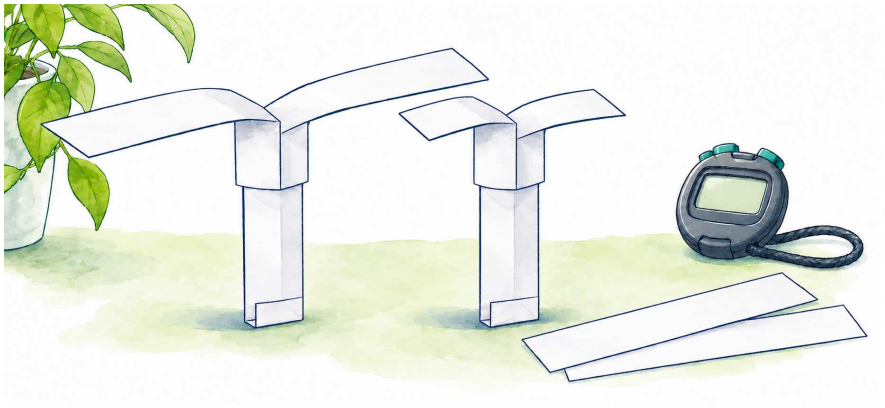
4. What is wrong with saying p=0.07 means a 7% probability the null is true?

5. How does a matching 95% t-interval support a two-sided 5% test?

6. What design supports causation and what supports generalization?

P02 | Does a design change increase flight time?

Course project • Meeting 45. For paper helicopters made by this class, does the long-wing design increase mean fall time relative to the short-wing design under the defined testing conditions?



Comparing two paper designs

Work in groups. Define two reproducible paper-helicopter templates that differ only in wing length. Hold paper type, body length, release height, and measurement method fixed. Each of 12 paper-and-builder blocks supplies one of each design; randomly assign the two templates to the two paper pieces within a block, then randomly assign release order. Average three releases for each helicopter, retaining the helicopter as the experimental unit. For analysis practice, an illustrative set of 12 block differences (long minus short, in seconds) is: -0.01, 0.04, 0.05, 0.07, 0.08, 0.10, 0.11, 0.12, 0.13, 0.15, 0.16, 0.20. These fallback data are fictional, not class findings.

1. On a separate planning sheet, write the investigative question and define the population to which you would like to apply the result. Identify treatments, experimental units, blocking variable, response, and a reproducible random-assignment procedure before collecting data.

2. Collect paired data under the agreed protocol, or use the labeled fallback differences. On a separate data sheet, record each block ID, each helicopter's three release times and average, and the long-minus-short difference. Explain below why three releases of one helicopter are repeated measurements rather than three independently assigned helicopters.

3. Plot the twelve paired differences. Report their sample mean and median to three decimal places, then describe spread and unusual features. Which center belongs in the paired t statistic? Use the difference plot to check its shape condition.

P02 | Analysis and reporting

4. Define the population mean difference, state the hypotheses for improvement, and conduct the paired t test at alpha=0.05. Show the standard error, t statistic, degrees of freedom, p-value, and a contextual conclusion.

5. Construct a 95% two-sided interval for the mean difference. Interpret its units and direction. Explain why its endpoints add information beyond a significance decision.

6. Identify what random assignment supports and what the convenience choice of paper, builders, and testing conditions limits. On separate pages, write the group brief with the complete method, graph, inference, limitations, and one proposed replication. Each student then writes a separate paragraph explaining why an independent two-sample test would answer this design poorly.

Required product

One group methods-and-results brief on separate pages, with the planning sheet, raw measurements, and paired differences attached, plus each student's individual explanation. State explicitly whether the analysis uses real class observations or the illustrative fallback dataset. For the fallback route, retain the supplied block differences; do not invent raw release times.

Assessment criteria

- 4 points: defined question/parameter, distinct treatments, experimental units and reproducible randomization.
- 4 points: paired design respected, correct analysis unit, labeled difference graph and conditions.
- 4 points: correct test setup, numerical execution, and p-value interpretation.
- 4 points: confidence interval and contextual conclusion with a justified scope of inference.
- 4 points: individual explanation, evidence of critique/revision, and a useful replication proposal.

Unit 10 | Two-Variable Data and Regression

Relationships between quantitative variables require both graphical and numerical reasoning. This unit begins with paired observations and scatterplots, then develops correlation, prediction, residuals, and least-squares regression. A recurring illustrative print-job dataset lets students connect different representations of the same relationship. Counterexamples show why a high correlation cannot establish linear form or causation. Students use technology to fit models while taking responsibility for variable roles, units, input range, and contextual interpretations. The final lesson explains the coefficient of determination through a mean-only prediction baseline. Residual standard deviation is identified as a supporting extension; inference for regression slopes is outside the revised required scope.

CED topics: 5.1, 5.2, 5.3, 5.4, 5.5

IN THIS UNIT

Lesson	Focus
S10.01	Seeing relationships in paired quantitative data
S10.02	Correlation: strength of a linear association
S10.03	Linear predictions and their limits
S10.04	Residuals and checking a linear model
S10.05	Least squares and explained variation

Keep a route's measurements together

Route	Elevation gain (m)	Walking time (min)
P	80	24
Q	125	32
R	170	38

Illustrative observations. Each row contributes one point to a scatterplot.

LOCAL QUESTION

Elevation and walking time may be related. What would a fitted line describe, and why would a strong association alone not establish causation?

S10.01 | Seeing relationships in paired quantitative data

CED 5.1

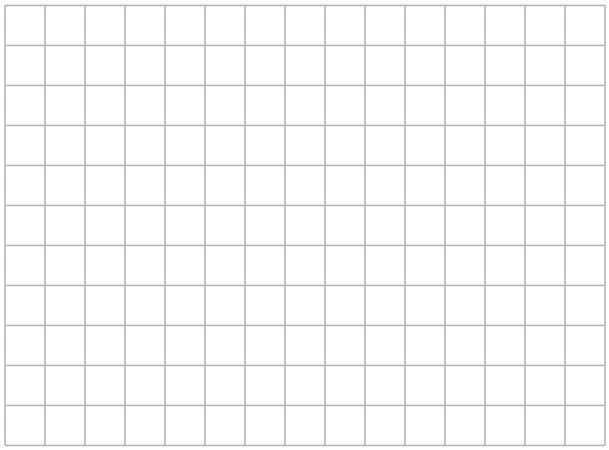
GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

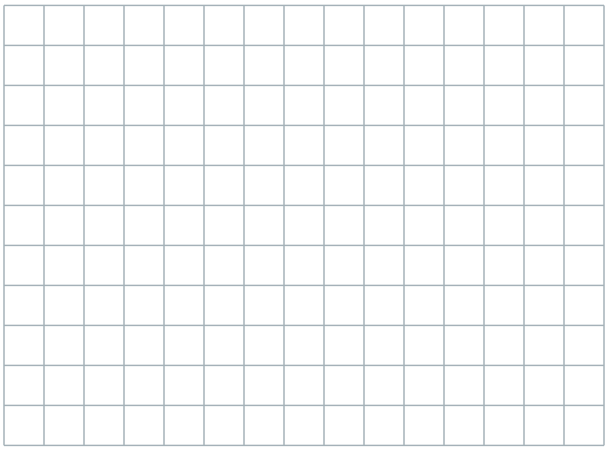
A relationship can be strong without being linear

Use two illustrative datasets with x=1,2,3,4,5. Dataset A has y=2,4,5,4,5. Dataset B has y=5,2,1,2,5.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Plot both datasets with the same axis scales.





2. Describe direction, form, strength, and unusual features for each.

3. Explain why “no straight-line trend” does not mean “no relationship.”

4. Suggest a prediction question for which a straight line might be misleading for B.

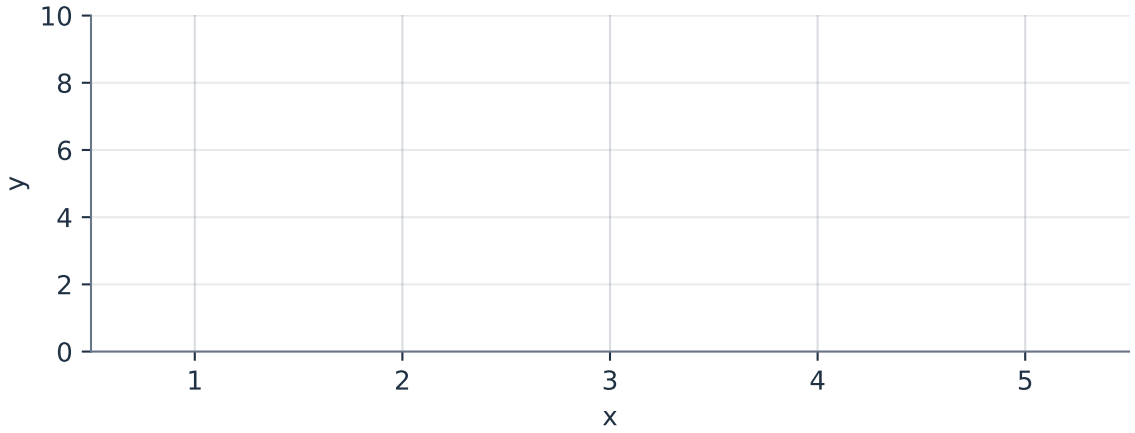
Discuss your results

- Why is a single upward or downward direction not an adequate description of dataset B?
- What would be lost if you reported only that B has no linear trend?

S10.01 | Practice and exit

INDEPENDENT APPLICATION

Construct the given scatterplot



Use the first independent-practice problem. Plot (1,9), (2,8), (3,6), (4,5), and (5,3). Describe direction, form, strength, and unusual features. These are mathematical pairs, not measured observations.

10

1. Use the supplied coordinate grid to plot the five pairs$(1,9),(2,8),(3,6),(4,5),(5,3)$. These are a mathematical example with no claimed real-world source. Label the axes x and y. Describe the direction, form, strength, and any unusual features.

2. One column of paired data is sorted alone. Explain the consequence.

3. A plot has a clear curved pattern with little scatter. Is “weak because it is nonlinear” appropriate?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

What does one dot represent in a scatterplot of a student’s commute distance and commute time?

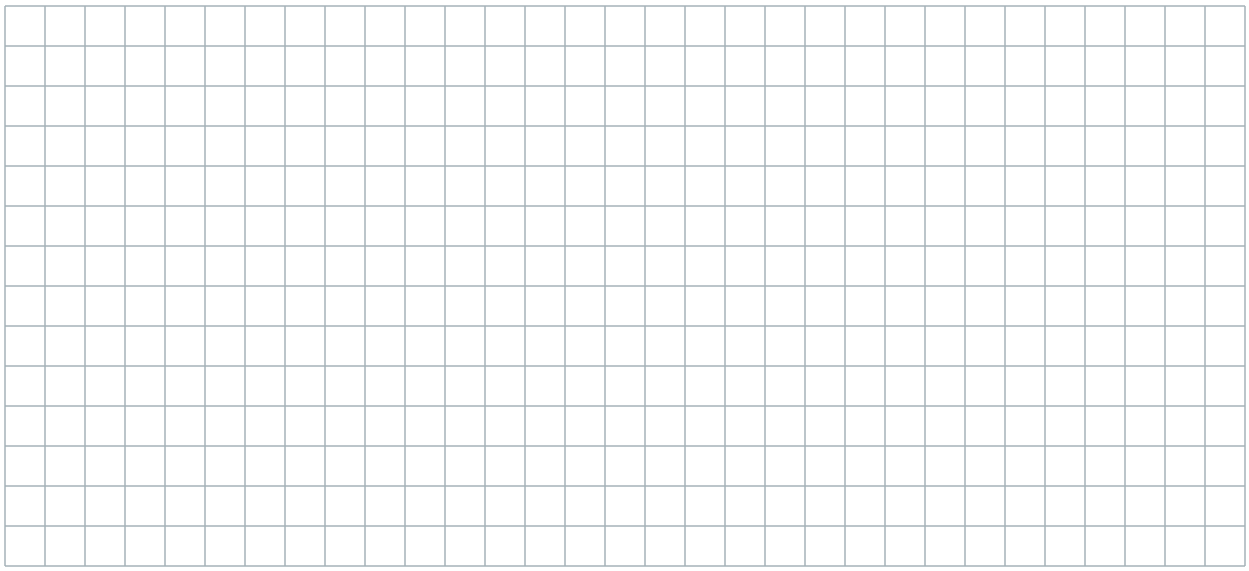
S10.01-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR48 for S10.02 and answer both checks.

1. OpenStax gives the age and vocabulary-size pairs$(3,655),(4,1098),(6,2463),(7,3195)$, where age is in years and vocabulary size is number of words. Construct a scatterplot with labeled axes. Describe the direction, form, and strength of the association in context.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 12.2, Example 12.5, pp.620-621 (PDF 632-633); data unchanged.



2. For pairs $(1,2),(2,3),(3,4),(4,5),(12,1)$, identify the unusual feature and explain why it should be investigated.

3. A school records grade level as $9,10,11,12$ and favorite lunch as $1,2,3$. Why is correlation not an appropriate first summary?

S10.01-HW | Continued

4. Two clusters follow different trends. A student describes only the combined plot. State what additional analysis is useful.

5. Cumulative: Explain how random sampling affects a descriptive scatterplot conclusion.

6. Write a valid statistical question about two quantitative variables that avoids a causal assumption, and identify axes.

S10.02 | Correlation: strength of a linear association

CED 5.2

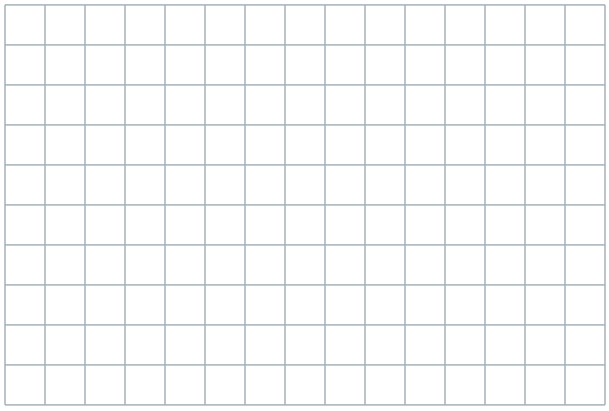
GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

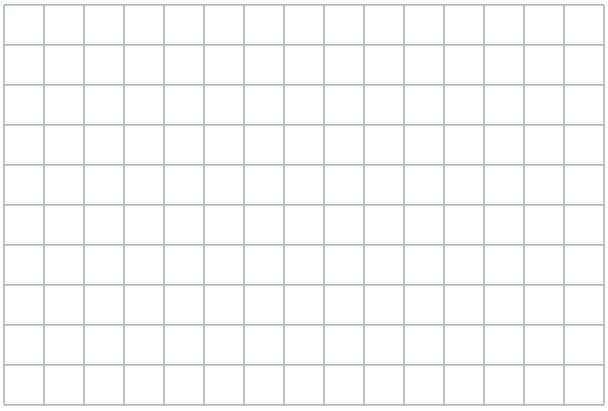
Can high r coexist with curvature?

Use illustrative x=(1,2,3,4,5), y=(1,4,9,16,25). Also use the U-shaped dataset x=(1,2,3,4,5), y=(5,2,1,2,5).

Materials. Graphing calculator or statistical software; pencil and paper.

1. Plot both datasets before calculating r.





2. Calculate r for each.

3. Explain why the first large r does not prove a straight-line model is adequate.

4. Explain why the second r does not mean the variables are unrelated.

Discuss your results

- What information do the scatterplots reveal that the two correlation values omit?
- Can changing the units remove the curvature in either plot? Explain.

S10.02 | Practice and exit

INDEPENDENT APPLICATION

Check transformations of correlation

Quantity	Given values / student work
x	1, 2, 3, 4, 5
y	2, 4, 5, 4, 5
Original r	_____
r after replacing y with −2y	_____
r after exchanging x and y	_____
Contextual interpretation	_____

Use the first independent-practice problem, where x is illustrative hours and y is output units. Complete the comparisons; calculated correlations are not supplied.

1. Use the supplied correlation table with $x=(1,2,3,4,5)$ hours and $y=(2,4,5,4,5)$ output units. Calculate r. Then predict and verify r after replacing every y by $-2y$ and after exchanging the roles of x and y. Interpret the original association.

2. A distance variable is converted from kilometers to meters. How does r change?

3. A student claims $r=0$ means independence. Correct the claim.

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

What happens to correlation if x and y are interchanged?

S10.02-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR49 for S10.03 and answer both checks.

1. Calculate r for x=(1,2,3,4,5), y=(2,4,5,4,5), then interpret it for illustrative hours and output units.

2. For the preceding data, multiply every y by −2. Predict the new r and justify.

3. A plot is clearly curved but has r=0.96. Explain why the graph should not be ignored.

S10.02-HW | Continued

4. A community has correlated ice-cream purchases and outdoor emergency calls. Give a possible common cause without claiming it is established.

5. Cumulative: What type of study would strengthen a causal comparison between two controllable task conditions?

6. The OpenStax exam-score data have correlation r≈0.66309. A student converts both exams to percent scores and reports that correlation must increase because the two exams now share the same scale. Is the student correct? Explain.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 12.3, Example 12.6, pp.623-630 (PDF 635-642); transformation question adapted.

S10.03 | Linear predictions and their limits

CED 5.3, 5.5

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Change units, retain the prediction

Use the illustrative model predicted seconds=9.4643+2.11905 x, where x is page count. The observed print jobs had 1-8 pages. Define m=completion time in minutes and z=page count in tens of pages, so z=x/10.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Rewrite the model to predict minutes from pages.

2. Rewrite it to predict seconds from tens of pages.

3. Use each equation to predict the same 6-page job.

4. Explain what changes in the slope interpretation and what does not change in the physical prediction.

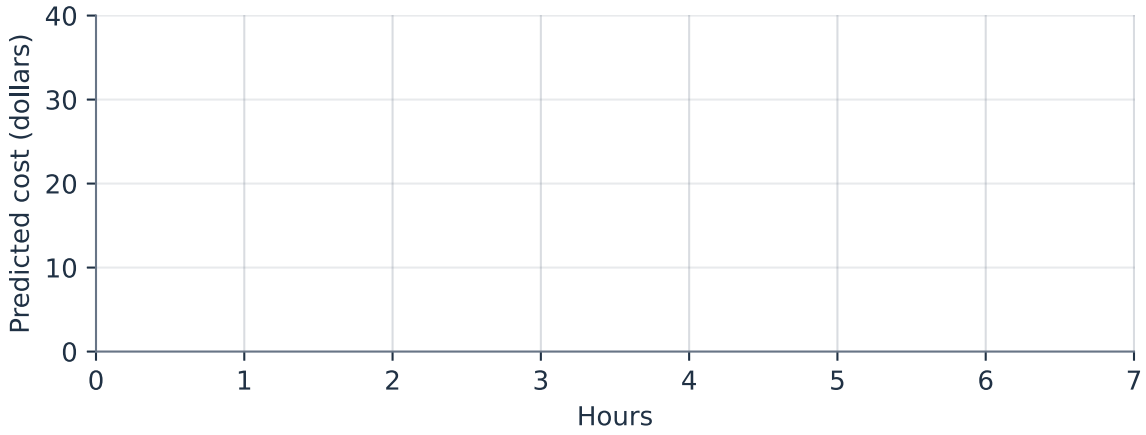
Discuss your results

- What is the observed input range in tens of pages?
- Why does expressing the model in minutes not make a 30-page prediction more reliable?

S10.03 | Practice and exit

INDEPENDENT APPLICATION

Translate a prediction equation into a graph



Given practice model: predicted cost = 12 + 3.5 × hours. Draw the line and mark the prediction for 5 hours; interpret the slope.

10

1. A model predicts cost in dollars as 12+3.5 x, where x is hours. Predict 5 hours and interpret slope. On the supplied axes, draw the prediction line and mark the prediction for 5 hours.

2. A model fitted for x from 20 to 60 is used at x=10. Classify the prediction and state its limitation.

3. A regression has a negative intercept for a response that cannot be negative. Must all in-range predictions be useless?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

In a model predicting seconds from pages, what are the slope units?

S10.03-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 6. **Optional practice.** 5. **Prepare for the next meeting.** Read PR50 for S10.04 and answer both checks.

1. For the OpenStax exam-score line, $\hat{y} = -173.51336 + 4.827394x$, predict the final-exam score for a third-exam score of 70. Interpret the slope in context. The observed third-exam range is 65-75 points.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 12.3, Example 12.6, pp.623-629 (PDF 635-641); input changed to 70.

2. Use the same exam-score model to predict a final-exam score for x=80. Explain why the numerical result should be treated cautiously.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 12.3, Example 12.6, pp.623-629 (PDF 635-641).

3. The final exam is scored out of 200 points. Rewrite the exam-score model to predict the final-exam percentage from the third-exam point score x. Explain how the slope changes.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Extension of Section 12.3, Example 12.6, pp.623-629 (PDF 635-641).

S10.03-HW | Continued

4. A student interprets a slope of −0.8 as “r=−0.8.” Explain why this is unsupported.

5. Cumulative: Why does a positive fitted slope from an observational study not establish causation?

6. A model predicts y from x. Explain why algebraically solving it for x does not generally give the least-squares model predicting x from y.

S10.04 | Residuals and checking a linear model

CED 5.4

OPENING PREDICTION

In the “Data and fitted line” panel of the reference graphic, compare the points at x = 1, 3, and 5 with the line. Where does the line predict too low or too high?

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

A high correlation can leave a curved residual pattern

Use illustrative $x=(1,2,3,4,5)$, $y=(1,4,9,16,25)$. A least-squares line for these pairs is predicted $y=-7+6x$.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Calculate predictions and residuals for all five points.

2. Plot residuals against x with a zero reference line.

3. Describe the pattern and assess the linear model.

4. Explain why $r\approx0.981$ does not overturn the residual evidence.

Discuss your results

- At which x values does the line underpredict, and at which does it overpredict?
- Why is the sign pattern more useful for checking linear form than the large correlation alone?

S10.04 | Practice and exit

INDEPENDENT APPLICATION

Calculate a residual for each given observation

x	Observed y	Predicted ŷ	Residual y − ŷ
1	8	_____	_____
2	8	_____	_____
3	12	_____	_____
4	12	_____	_____

Use the first independent-practice problem and the supplied model ŷ = 5 + 2x. Fill the final two columns.

1. Complete the supplied prediction-and-residual table for the model $\hat{y} = 5 + 2x$ and the four pairs $(1,8),(2,8),(3,12),(4,12)$. Show observed minus predicted for each residual. Explain which observations the model overpredicts.

2. A residual is −4 and the predicted score is 70. Find the observed score.

3. Residuals form an arch above zero in the middle and below zero at the ends. What does that suggest?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

What should a suitable linear-model residual plot look like?

S10.04-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 4, 5, 6. **Optional practice.** 3. **Prepare for the next meeting.** Read PR51 for S10.05 and answer both checks.

1. For predicted y=5+2 x and observed pairs $(1,8),(2,8),(3,12),(4,12)$, calculate all residuals.

2. Plot those residuals and state what can be said from only four observations.

3. One student in the OpenStax data has third-exam score 67 and final-exam score 133. Use $\hat{y} = -173.51336 + 4.827394x$ to calculate the residual. Interpret its sign and magnitude in context.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 12.3, Example 12.6, Table 12.3, pp.623-625 (PDF 635-637); residual question adapted.

S10.04-HW | Continued

4. A residual plot shows no curve but errors as large as 30 minutes. Is the model necessarily useful for scheduling within 5 minutes?

5. Cumulative: A model has r=0.98 and a strong U-shaped residual plot. Which evidence addresses linear adequacy more directly?

6. A point has a very unusual x value but residual nearly 0. Must it be a large vertical outlier? Explain.

S10.05 | Least squares and explained variation

CED 5.5, 5.4, 5.2

GUIDED APPLICATION | AFTER THE WORKED EXAMPLE

Which line is best by the stated criterion?

Use the same illustrative pairs with x measured in half-hours: x = (2, 4, 6, 8, 10) and y = (2, 4, 5, 4, 5). Compare candidate lines A: predicted y = 4, B: predicted y = 1 + 0.50x, and C: predicted y = 2.2 + 0.30x.

Materials. Graphing calculator or statistical software; pencil and paper.

1. Compute residuals and SSE for each candidate.

2. Rank the lines using the least-squares criterion.

3. Check whether the selected line passes through the mean point.

4. Explain why a zero sum of residuals would not be enough to choose the best line.

Discuss your results

- Why can all three candidate lines have residuals that sum to zero yet have different SSEs?
- What is the baseline prediction used when interpreting r^2 for this dataset?

S10.05 | Practice and exit

INDEPENDENT APPLICATION

Account for the variation

Quantity	Given / student work
Total variation, SST	200 seconds^2
Unexplained variation, SSE	50 seconds^2
Explained variation, SST − SSE (seconds^2)	_____
Explained fraction, r^2	_____
Interpretation of explained fraction	_____

Use the second independent-practice problem. Complete the missing quantities and write a careful interpretation.

1. A line has $r=-0.8$. Find r^2 and interpret it for predicting completion time from experience.

10

2. For a model predicting print-job completion time from page count, SST=200 and SSE=50 in squared seconds. Complete the variation organizer. Calculate r^2 and interpret it in context.

3. A fitted line is predicted $y=10+2x$ and $\bar{x}=4$. What $\bar{y}$ must hold for an ordinary least-squares line with an intercept?

Compare and revise. Compare explanations and improve one answer above.

Exit check | Work on your own

What does $r^2=0.81$ mean, and what does it not mean?

S10.05-HW | Nightly practice

Name _____ Date _____

Required problems. 1, 2, 3, 4, 5. **Optional practice.** 6. **Prepare for the next meeting.** Read PR52 for C6 and answer both checks.

1. Fit a least-squares line to the illustrative print pairs x=1,2,3,4,5,6,7,8 and y=12,15,14,18,20,21,25,27. Report coefficients and r^2.

2. For the print data, verify that the fitted line passes through the mean point.

3. For the OpenStax exam-score data, SST=4326.727273 and SSE=2424.295100, in squared final-exam points. Calculate r^2 and interpret it in context.

Source: Barbara Illowsky and Susan Dean, OpenStax, Introductory Statistics 2e, supplied 2026 printing. Section 12.3, Example 12.6 and coefficient-of-determination discussion, pp.623-631 (PDF 635-643).

S10.05-HW | Continued

4. Explain why the line connecting the first and last observed points need not minimize SSE.

5. Cumulative: A line has high r^2 but a U-shaped residual plot. Should it be accepted as an adequate linear model?

6. Optional supporting extension (not required AP core): For n=8 and SSE=7.404762 seconds squared, find residual standard deviation and interpret its units.

Unit 10 | Cumulative synthesis

1. Describe the four features of a scatterplot that should be considered.

2. Can r=0 coexist with a strong relationship?

3. For predicted y=4+3 x, x=2, and observed y=12, find prediction and residual.

4. What does a U-shaped residual plot indicate?

5. A model has SSE=15 and SST=60. Find and interpret r^2.

6. What does a high r^2 not establish?