FORMULA PRACTICE

Fold-over recall cards

Print single-sided at actual size. Trim the outer rectangle first, cut the five strips, and fold on the dashed center line with the printed sides out. Tape or glue the unprinted sides together.

01 / RADIANS AND TRIGONOMETRY

How do you convert degrees to radians? How is arc length related to angle?

01 / ANSWER

Multiply degrees by $\frac{\pi}{180}$. An angle of θ radians cuts off arc length $s = r\theta$ on a circle of radius r (use positive θ for length).

02 / RADIANS AND TRIGONOMETRY

On the unit circle, what are the coordinates of the point at angle t? How do you find signs outside the first quadrant?

02 / ANSWER

The point is $(\cos t, \sin t)$. The horizontal coordinate gives the sign of cosine. The vertical coordinate gives the sign of sine.

03 / RADIANS AND TRIGONOMETRY

Give the sines of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ in order.

03 / ANSWER

$0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$. On the circle, these are heights.

04 / RADIANS AND TRIGONOMETRY

Give the cosines of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ in order.

04 / ANSWER

$1, \frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}, 0$. On the circle, these are horizontal coordinates.

05 / RADIANS AND TRIGONOMETRY

How do you rebuild tangent, cotangent, secant, and cosecant from sine and cosine?

05 / ANSWER

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$. Each denominator must be nonzero.

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06 / RADIANS AND TRIGONOMETRY State the Pythagorean trig identity and its two quotient forms.	**06 / ANSWER** $\sin^2 x + \cos^2 x = 1$. Divide by $\cos^2 x$ for $1 + \tan^2 x = \sec^2 x$, or by $\sin^2 x$ for $1 + \cot^2 x = \csc^2 x$, where defined.
07 / RADIANS AND TRIGONOMETRY How can you rewrite a product of sine and cosine of the same angle? What is the cosine double-angle identity?	**07 / ANSWER** $2 \sin x \cos x = \sin(2x)$ and $\cos(2x) = \cos^2 x - \sin^2 x$.
08 / GEOMETRY FOR CALCULUS For a circle of radius r, give circumference and area.	**08 / ANSWER** $C = 2\pi r$ and $A = \pi r^2$. Diameter $d = 2r$.
09 / GEOMETRY FOR CALCULUS For a right triangle, how do you relate lengths? How do you find the area of any triangle?	**09 / ANSWER** $a^2 + b^2 = c^2$, where c is the hypotenuse. For any triangle, $A = \frac{1}{2}bh$, with h perpendicular to the base.
10 / GEOMETRY FOR CALCULUS A cylindrical tank has radius r and height h. Give volume and closed surface area.	**10 / ANSWER** $V = \pi r^2 h$. Two disks and the side give $S = 2\pi r^2 + 2\pi r h$. Omit one disk for an open top.

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11 / GEOMETRY FOR CALCULUS

What is a cone's volume? In a cone with its tip at the bottom, how do water radius and depth relate?

11 / ANSWER

$V = \frac{1}{3}\pi r^2 h$. For water depth $h > 0$ above the tip, $r/h = R/H$ by similar triangles. Full-cone radius and height are R, H.

12 / GEOMETRY FOR CALCULUS

For a sphere of radius r, give volume and surface area.

12 / ANSWER

$V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$.

13 / GEOMETRY FOR CALCULUS

An interval of length w is the side of a square or equilateral triangle, or the diameter of a semicircle. Give each cross-sectional area.

13 / ANSWER

Square w^2. Equilateral triangle $\frac{\sqrt{3}}{4}w^2$. Semicircle $\frac{\pi}{8}w^2$ since its radius is $\frac{w}{2}$.

14 / GEOMETRY FOR CALCULUS

What area do you integrate for a disk or washer perpendicular to the axis of rotation?

14 / ANSWER

$A = \pi\left(R^2 - r^2\right)$, with R and r the outer and inner distances to the axis. A disk has $r = 0$.

15 / DIFFERENTIATION RULES

What are the constant and power rules? How do you differentiate a sum?

15 / ANSWER

$\frac{d}{dx}c = 0$ and $\frac{d}{dx}x^n = nx^{n-1}$ where x^n is real and differentiable. For example, $\left(x^{-2}\right)' = -2x^{-3}$ requires $x \neq 0$. Differentiate sums term by term.

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16 / DIFFERENTIATION RULES

Give the product and quotient rules for differentiable u, v.

16 / ANSWER

$(uv)' = u'v + uv'$. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$.

17 / DIFFERENTIATION RULES

Give the chain rule for a composite function.

17 / ANSWER

If $y = f\left(g(x)\right)$, then $y' = f'\left(g(x)\right)g'(x)$, provided both derivatives exist at the relevant inputs.

18 / DIFFERENTIATION RULES

Differentiate e^u and a^u when $u = u(x)$.

18 / ANSWER

$(e^u)' = e^u u'$. $(a^u)' = a^u \ln(a)\, u'$ for constant $a > 0$.

19 / DIFFERENTIATION RULES

Differentiate $\ln|u|$ and $\log_a u$ when $u = u(x)$.

19 / ANSWER

$(\ln|u|)' = \frac{u'}{u}$ for $u \neq 0$. $\left(\log_a u\right)' = \frac{u'}{u \ln a}$ for $u > 0$, $a > 0$, $a \neq 1$.

20 / DIFFERENTIATION RULES

How do you obtain an inverse function's derivative from a matching point on the original graph?

20 / ANSWER

If $f(a) = b$ and f has a local differentiable inverse with $f'(a) \neq 0$, then $\left(f^{-1}\right)'(b) = \frac{1}{f'(a)}$.

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21 / DIFFERENTIATION RULES Give the tangent-line approximation near $x = a$.	**21 / ANSWER** $L(x) = f(a) + f'(a)(x - a)$. Use $L(x)$ to estimate $f(x)$ close to a when f is differentiable there.
22 / TRIG AND INVERSE-TRIG DERIVATIVES What are the sine and cosine derivatives?	**22 / ANSWER** Angles are in radians. $(\sin x)' = \cos x$ and $(\cos x)' = -\sin x$.
23 / TRIG AND INVERSE-TRIG DERIVATIVES What are the tangent and cotangent derivatives?	**23 / ANSWER** Angles are in radians. $(\tan x)' = \sec^2 x$ and $(\cot x)' = -\csc^2 x$, wherever the original functions are defined.
24 / TRIG AND INVERSE-TRIG DERIVATIVES What are the secant and cosecant derivatives?	**24 / ANSWER** Angles are in radians. $(\sec x)' = \sec x \tan x$ and $(\csc x)' = -\csc x \cot x$, wherever the functions are defined.
25 / TRIG AND INVERSE-TRIG DERIVATIVES Differentiate $\arcsin u$ and $\arccos u$.	**25 / ANSWER** Inverse-trig outputs are in radians. $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$ and $(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$ for $\lvert u \rvert < 1$.

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26 / TRIG AND INVERSE-TRIG DERIVATIVES

Differentiate $\arctan u$.

26 / ANSWER

Inverse-trig outputs are in radians. $(\arctan u)' = \frac{u'}{1+u^2}$ for every real u where u' exists.

27 / COMMON ANTIDERIVATIVES

Find $\int x^n\,dx$. Which exponent requires a different rule?

27 / ANSWER

For $n \neq -1$, use $\frac{x^{n+1}}{n+1} + C$ on an interval where the powers are defined. For $n = -1$, use $\ln|x| + C$ on an interval avoiding zero.

28 / COMMON ANTIDERIVATIVES

Find $\int e^x\,dx$ and $\int a^x\,dx$.

28 / ANSWER

$e^x + C$ and $\frac{a^x}{\ln a} + C$, respectively, for $a > 0$, $a \neq 1$ in the second formula.

29 / COMMON ANTIDERIVATIVES

Find $\int \sin x\,dx$ and $\int \cos x\,dx$.

29 / ANSWER

Angles are in radians. $-\cos x + C$ and $\sin x + C$, respectively.

30 / COMMON ANTIDERIVATIVES

Find $\int \sec^2 x\,dx$ and $\int \csc^2 x\,dx$.

30 / ANSWER

Angles are in radians. $\tan x + C$ and $-\cot x + C$, respectively, on intervals where the integrands are defined.

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31 / COMMON ANTIDERIVATIVES

Find $\int \sec x \tan x \, dx$ and $\int \csc x \cot x \, dx$.

31 / ANSWER

Angles are in radians. $\sec x + C$ and $-\csc x + C$, respectively, on intervals where the integrands are defined.

32 / COMMON ANTIDERIVATIVES

Find $\int \frac{1}{1+x^2} \, dx$ and $\int \frac{1}{\sqrt{1-x^2}} \, dx$.

32 / ANSWER

Inverse-trig outputs are in radians. $\arctan x + C$ for all real x and $\arcsin x + C$ for $|x| < 1$, respectively.

33 / COMMON ANTIDERIVATIVES

Find $\int \tan x \, dx$ and $\int \cot x \, dx$.

33 / ANSWER

Angles are in radians. $-\ln|\cos x| + C$ and $\ln|\sin x| + C$, respectively, on intervals where the integrands are defined.

34 / ACCUMULATION AND SUBSTITUTION

How do you evaluate a definite integral using an antiderivative?

34 / ANSWER

If f is continuous on $[a, b]$ and $F' = f$, then $\int_a^b f(x) \, dx = F(b) - F(a)$.

35 / ACCUMULATION AND SUBSTITUTION

If $H(x) = \int_a^{g(x)} f(t) \, dt$, what is $H'(x)$?

35 / ANSWER

$H'(x) = f\big(g(x)\big)g'(x)$ for continuous f and differentiable g. If $g(x)$ is the lower endpoint and the upper endpoint is fixed, reverse the sign.

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36 / ACCUMULATION AND SUBSTITUTION How do the bounds change under the substitution $u = g(x)$?	**36 / ANSWER** With $du = g'(x)\,dx$, use $u = g(a)$ and $u = g(b)$ as the new bounds. Thus $\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$ for continuous f and continuously differentiable g.
37 / ACCUMULATION AND SUBSTITUTION How do you distinguish average value from average rate of change?	**37 / ANSWER** On $[a, b]$ with $a < b$, average value is $\frac{1}{b-a}\int_a^b f(x)\,dx$ and has the units of f. Average rate of change is $\frac{f(b)-f(a)}{b-a}$ and has units of f divided by units of x.
38 / ACCUMULATION AND SUBSTITUTION How do displacement and total distance come from velocity?	**38 / ANSWER** On $a \le t \le b$, displacement is $\int_a^b v(t)\,dt$. Distance is $\int_a^b \lvert v(t)\rvert\,dt$. Split the interval where velocity changes sign if integrating by hand.
39 / THEOREMS AND THEIR CONDITIONS When does the Intermediate Value Theorem guarantee a solution to $f(x) = k$?	**39 / ANSWER** If f is continuous on $[a, b]$ and k lies strictly between $f(a)$ and $f(b)$, then some c in (a, b) has $f(c) = k$.
40 / THEOREMS AND THEIR CONDITIONS When are both absolute extrema guaranteed? Where do you look for their values?	**40 / ANSWER** Continuous f on closed $[a, b]$ attains both. Compare f at each endpoint and at interior critical numbers, where $f' = 0$ or f' fails to exist.

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41 / THEOREMS AND THEIR CONDITIONS State the conditions and conclusion of the Mean Value Theorem.	**41 / ANSWER** If f is continuous on $[a,b]$ and differentiable on (a,b), some c in (a,b) satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$.
42 / THEOREMS AND THEIR CONDITIONS When can you apply L'Hôpital's rule to a quotient limit?	**42 / ANSWER** First verify a form $0/0$ or ∞/∞. On the approach interval, both functions must be differentiable with $g' \neq 0$. If $\lim \frac{f'}{g'}$ exists (finite or infinite), it gives $\lim \frac{f}{g}$.
ADD A FORMULA YOU MISSED	**ANSWER AND CONDITIONS**
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