Calculus AB
Student workbook

Use your completed diagnostic and feedback to choose the next skill. Attempt an assigned fresh problem before reading its model, then open the matching topic in the reference.

How to use these pages

Start with the assigned repair clinic. Follow your tutor's weekly assignment strip. Read the assigned reference models, attempt the assigned C checks, then use the matching P numbers here. Some topics have P3 or later problems. Use mixed work when you can select the method independently.

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PREREQUISITE REPAIR / R1

Factoring and canceling

Use when substitution gives a zero denominator and a numerator with a common factor. Keep the restrictions from the original expression.

Worked model

Simplify $(x^2 - x - 6)/(x - 3)$. Factoring the numerator gives $(x - 3)(x + 2)$. For $x \neq 3$ the quotient equals $x + 2$. Nearby values therefore approach 5 as x approaches 3, although the original quotient has no value at 3. For a difference quotient, subtract complete outputs before canceling. If $F(x) = x^2 - 2x$, then $F(3 + h) = (3 + h)^2 - 2(3 + h) = 3 + 4h + h^2$ and $F(3) = 3$. Hence $[F(3 + h) - F(3)]/h = [(3 + 4h + h^2) - 3]/h = 4 + h$ for $h \neq 0$.

TRY TOGETHER

Simplify $(x^2 - 9)/(x^2 + x - 6)$. Which inputs were excluded before you canceled?

TRY INDEPENDENTLY

For a fixed number a, expand the numerator and simplify $[(a + h)^2 - a^2]/h$. State the restriction on h.

RETURN IN 2–4 DAYS

Simplify $(x^3 - 8)/(x - 2)$ and find the value the quotient approaches near 2.

PREREQUISITE REPAIR / R2

Radicals and conjugates

Use a conjugate when the numerator is a difference of square roots. The product of a sum and a difference removes the radicals from that product.

Worked model

For $h \neq 0$, multiply $\left(\sqrt{9+h}-3\right)/h$ by $\left(\sqrt{9+h}+3\right)/\left(\sqrt{9+h}+3\right)$. The numerator becomes $(9+h)-9=h$. Cancel to obtain $1/\left(\sqrt{9+h}+3\right)$, valid for $h \geq -9$ and $h \neq 0$. Near 0 it approaches $1/6$.

TRY TOGETHER

Rationalize $\left(\sqrt{x+5}-3\right)/(x-4)$ and evaluate the value it approaches near $x=4$.

TRY INDEPENDENTLY

Simplify $\left(\sqrt{1+3h}-1\right)/h$ and find its limiting value near zero.

RETURN IN 2–4 DAYS

Rationalize $\left(\sqrt{x+4}-\sqrt{2x+1}\right)/(x-3)$. What value does it approach near 3?

PREREQUISITE REPAIR / R3

Composition and domain

Read compositions from the inside out. Check the domain of the inner rule and then ask which of its outputs the outer rule accepts.

Worked model

Let $f(u) = \sqrt{4-u}$ and $g(t) = t^2$. In $f\big(g(t)\big)$ the outer rule receives t^2, so the result is $\sqrt{4-t^2}$. The real square root requires $4 - t^2 \geq 0$, hence $-2 \leq t \leq 2$.

TRY TOGETHER

Using the same f and g, write $g(f(t))$ and find its domain. Evaluate it at $t = -5$.

TRY INDEPENDENTLY

Let $L(u) = \ln u$ and $q(x) = x^2 - 9$. Write $L(q(x))$ and find all allowed real inputs.

RETURN IN 2–4 DAYS

Let $F(u) = 1/(u-2)$ and $g(x) = 1 + \sqrt{x}$. Find $F(g(x))$, its domain, and its value at 4.

PREREQUISITE REPAIR / R4

Graphs, secant slopes, and units

Identify the meaning of each axis before dividing changes. An exact interval slope and an estimate of the slope at one instant are different claims.

Worked model

A position graph contains $(2, 11)$ and $(5, 5)$, with time in seconds and position in meters. The secant slope is $(5 - 11)/(5 - 2) = -2$ m/s. Position decreases by 2 meters per second on average during that interval. The two points alone cannot determine the speed at 3 s.

TRY TOGETHER

A distance-from-school graph joins $(0, 4)$, $(2, 10)$, and $(5, 7)$ by straight segments. Time is in minutes and distance is in kilometers. Find the slope of each segment and interpret the second one.

TRY INDEPENDENTLY

A tank contains 18, 24, and 20 L at 0, 2, and 5 min, respectively. Compute its average rate of change over the full interval and include units.

RETURN IN 2–4 DAYS

A smooth temperature trace reads 19.4, 20.0, and 20.4 degrees Celsius at 9, 10, and 11 min. Estimate the instantaneous rate at 10 min using the two surrounding readings. Explain why it is an estimate.

PREREQUISITE REPAIR / R5

Powers, exponentials, and logarithms

Use roots to interpret fractional powers, reciprocals for negative powers, and logarithms to solve when the unknown appears in an exponent.

Worked model

A mass of $5 \cdot 2^{t/3}$ grams reaches 30 grams when $2^{t/3} = 6$. Taking natural logs gives $\left(t/3\right) \ln 2 = \ln 6$, so $t = 3 \ln 6 / \ln 2$. If t is measured in hours, this is between 6 and 9 h because 6 lies between 2^2 and 2^3.

TRY TOGETHER

Solve $12e^{-0.4t} = 3$ for t. Give an exact answer and explain why the time is positive.

TRY INDEPENDENTLY

Rewrite $x^{-3/2}$ using a square root and positive powers. Give its real domain and evaluate at 9.

RETURN IN 2–4 DAYS

For $a, b > 0$, expand $\ln\left(a^2/\sqrt{b}\right)$ as a sum or difference of logarithms. Check the formula using $a = e^2$ and $b = e^4$.

PREREQUISITE REPAIR / R6

Radians and the unit circle

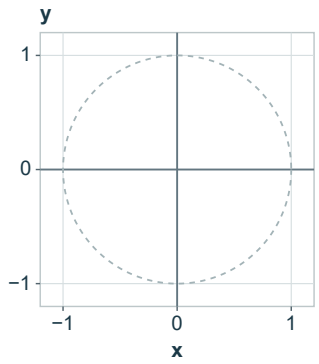
On the unit circle the coordinates are (cosine, sine). Use the reference angle for magnitudes and the quadrant for signs.

Worked model

Convert 150° to radians by multiplying by $\pi/180^\circ$: $\theta = 5\pi/6$. The reference angle is $\pi/6$. Quadrant II has a negative horizontal and positive vertical coordinate, so $(\cos\theta, \sin\theta) = \left(-\sqrt{3}/2, 1/2\right)$.

TRY TOGETHER

Convert 225° to radians. Draw its terminal ray and give exact values of sine, cosine, and tangent.



TRY INDEPENDENTLY

Find every θ in $[0, 2\pi)$ satisfying $\sin\theta = -\sqrt{3}/2$. Explain how you identified both angles.

RETURN IN 2–4 DAYS

On $[-\pi, \pi]$, find all solutions of $\cos\theta = 1/2$. For each, give its sine and angle in degrees.

PREREQUISITE REPAIR / R7

Geometry, similar triangles, and units

Draw the whole figure and the changing part. Match corresponding lengths before using an area or volume formula.

Worked model

A cone points downward and has height 12 cm and rim radius 4 cm. At water depth h the surface radius r obeys $r/h = 4/12$ by similar triangles. Substituting $r = h/3$ into $V = \pi r^2 h/3$ gives $V = \pi h^3/27$. For $h = 6$ cm this is 8π cubic centimeters.

TRY TOGETHER

A different cone points downward, with height 10 cm and rim diameter 6 cm. At depth 4 cm, find the surface radius and water volume.

TRY INDEPENDENTLY

A semicircular cross section has diameter 8 cm. Find its area and the volume of a slice that keeps this cross section constant for a thickness of 0.2 cm.

PREREQUISITE REPAIR / DELAYED RETRIEVAL

Clinic R7, continued

RETURN IN 2–4 DAYS

A point light atop a 12-foot pole shines past a 5-foot person standing 14 feet from the pole on level ground. Find the person's shadow length, measured from their feet to its tip.

PREREQUISITE REPAIR / R8

Rearranging equations and fractions

Identify the target variable and denominators that must stay nonzero. Clear denominators or factor the target before substituting numbers.

Worked model

To solve $1/p + 1/q = 1/f$ for q, subtract $1/p$: $1/q = (p - f)/(fp)$. Taking reciprocals gives $q = fp/(p - f)$. The solution requires $p, f \neq 0$ and $p \neq f$. If $p = f$, the equation would require $1/q = 0$, which has no finite solution.

TRY TOGETHER

Solve $3/y = 2 - 1/x$ for y. State all excluded x values and verify your rule at $x = 2$.

TRY INDEPENDENTLY

In the equation $2xu + 2yv = 0$, solve for v when $y \neq 0$. Evaluate your result when $x = 3$, $y = 4$, and $u = -2$.

RETURN IN 2–4 DAYS

Simplify $[1/(x + h) - 1/x]/h$. List every restriction from the original expression.

OPTIONAL PREREQUISITE FOLLOW-UP

Three skills to check when needed

Your tutor will select a question when this prerequisite needs a closer look.

PF1

For nonzero real u and v, solve $1/u + 1/v = 1/6$ for v. State all restrictions on u.

PF2

Rewrite $(2x - 1)^{-3/2}$ with a radical and no negative exponent. Give its real domain, then evaluate it at $x = 5 / 2$.

PF3

An inverted cone has full height 12 cm and full radius 5 cm. Draw a labeled axial section containing a water cone of depth h and radius r. Derive r in terms of h and then the water volume as a function of h. State the physical interval.

LESSON DIRECTORY

Find your workbook pages

LESSON	HOME LESSON	CED TOPICS	START
01	Algebra, radians, and geometry	Prerequisites	13
02	Limits and continuity	1.1–1.16	15
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08	Constraints, optimal values, and curves	5.10, 5.11, 5.12	79
09	From rate samples to a definite integral	6.1, 6.2, 6.3, 6.6	84
10	Accumulation and the fundamental theorem	6.4, 6.5, 6.7	90
11	Choosing an antiderivative	6.8, 6.9, 6.10, 6.14	95
12	Differential equations and slope fields	7.1–7.4, 7.6–7.8	101
13	Net change, average value, and motion	8.1, 8.2, 8.3	110
14	Area between curves	8.4, 8.5, 8.6	115
15	Volumes by slicing	8.7–8.12	120

When you return to a skill. Use the topic ID in the reference. P numbers identify workbook problems, including P3 and later items where present. The tutor topic directory lists every P number and its page. Use the assigned C and P numbers for each objective.

LESSON 01 / WITH A PARTNER

Algebra, radians, and geometry

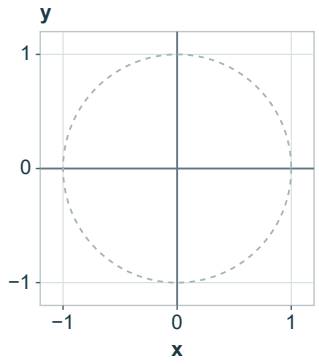
Work together on questions 1 and 2. Close the reference for question 3.

01.1 / TOGETHER

Simplify $\left(x^2 + x - 6\right) / \left(x + 3\right)$, state its domain, and locate any hole in its graph.

01.2 / TOGETHER

Sketch the terminal ray for $7\pi/6$, then give its sine, cosine, and tangent in exact form.



01.3 / SOLO

Let $g\left(x\right) = \ln\left(4 - x^2\right)$. Give its real domain and solve $g\left(x\right) = \ln 3$ exactly. State what makes exponentiating both sides legitimate.

LESSON 01 / WORK INDEPENDENTLY

Algebra, radians, and geometry

Show your reasoning. Finish the exit problem before checking the key.

01.4 / SOLO

A cone-shaped cup is 9 cm tall with a top radius of 6 cm. Water fills a similar cone of depth h and radius r. Draw an axial section, derive the relation between r and h, and express water volume as a function of h alone. Give its physical domain.

01.5 / MIXED

A player runs along a circular arc of radius 9 m through $80°$. Find the distance she runs and the area of the sector she sweeps out.

01.6 / EXIT

For $x \geq 0$, rewrite $\left(\sqrt{x} - 2\right) / (x - 4)$ so that you can describe its values near 4. State whether the original expression has a value at 4. If diagnostic A2 was unsure or unfamiliar, complete clinic R2 before attempting this exit.

LESSON 02 / CED 1.1

Change at an instant

Reference topic 1.1 • CHA-1.A

1.1 / P1

A skier's position is $s(t) = 20t - t^2$ meters. Compute the average velocities from 3 to 3.2 s and from 2.8 to 3 s. Use $s(3 + h)$ to find the exact instantaneous velocity at 3 s.

1.1 / P2

A thermometer reports 18.4, 18.9, and 19.2 °C at 9.5, 10.0, and 10.5 minutes. Estimate the temperature's rate of change at 10.0 minutes. Explain how another measurement could improve the estimate.

LESSON 02 / CED 1.2

Reading and writing limit notation

Reference topic 1.2 • LIM-1.A / LIM-1.B

1.2 / P1

Incoming measurements approach 12 from the left of $t = 5$ and 14 from the right. The reading at exactly 5 is 13. Write three statements in symbols, then decide whether the two-sided limit exists.

1.2 / P2

Write a formula for a function that satisfies $\lim_{x \to 1} f(x) = -2$ with $f(1) = 7$. Describe its graph near 1.

LESSON 02 / CED 1.2

Reading and writing limit notation

Reference topic 1.2 • LIM-1.A / LIM-1.B

1.2 / P3

A technician says a sensor output approaches 9 from either side of the input $u = -1$, although its output at -1 is 11. Express both claims in symbols, then read each limit number back in context.

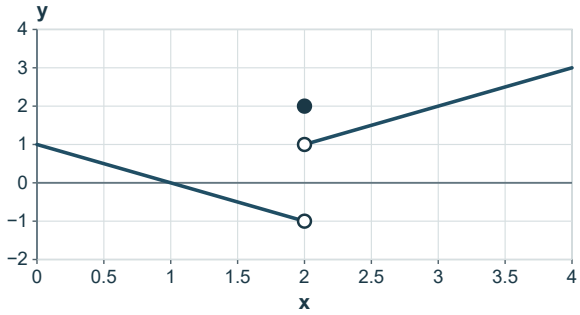
LESSON 02 / CED 1.3

Estimating limits from graphs

Reference topic 1.3 • LIM-1.C

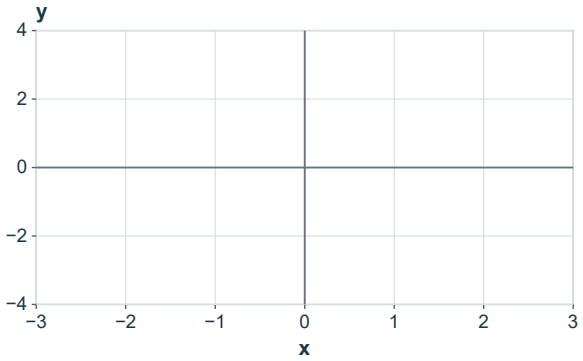
1.3 / P1

Read the left limit, the right limit, and $p(2)$ from the graph. Decide what can be said about $\lim_{x \to 2} p(x)$.



1.3 / P2

Sketch a function with equal left and right limits of -1 at $x = 0$ and with $q(0) = 2$. Label the open and filled points.



LESSON 02 / CED 1.4

Estimating limits from tables

Reference topic 1.4 • LIM-1.C

1.4 / P1

Estimate both one-sided limits and the two-sided limit at 2.

x	1.9	1.99	2	2.01	2.1
$f(x)$	-0.81	-0.9801	5	-1.0201	-1.21

1.4 / P2

Use $g(x) = \left(\sqrt{1+x} - 1\right)/x$ at $x = -0.1, -0.01, 0.01, 0.1$. Record values to four decimal places and estimate the limit at zero.

x	-0.1	-0.01	0.01	0.1
$g(x)$				

LESSON 02 / CED 1.5

Using limit laws

Reference topic 1.5 • LIM-1.D

1.5 / P1

As $x \to -1^{+}$, $u(x) \to 5$ and $v(x) \to 1$. Find the limit of $\sqrt{u(x) - v(x)}\,[u(x) + v(x)]/v(x)$ and justify each potentially restricted operation.

1.5 / P2

For $t < 2$ a function w equals $3 - t$. For $t \geq 2$ it equals $t - 1$. Decide whether $\lim_{t \to 2} e^{w(t)}$ exists and explain your reasoning.

LESSON 02 / CED 1.6

Rewriting expressions before taking limits

Reference topic 1.6 • LIM-1.E

1.6 / P1

Evaluate $\lim_{x \to 3} \left(x^3 - 27\right) / \left(x^2 - 9\right)$. Keep the restriction introduced when you cancel.

1.6 / P2

Evaluate $\lim_{x \to 0} \left(1 - \cos x\right) / \sin^2 x$ using multiplication by a conjugate.

LESSON 02 / CED 1.7

Choosing a limit procedure

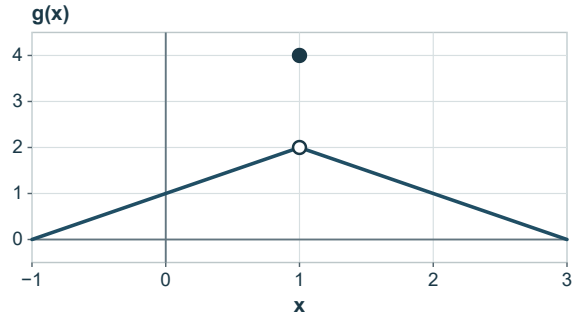
Reference topic 1.7 • Skill 1.C

1.7 / P1

Choose a method for each limit at $x = 1$, then compute: (a) $\left(x^2 + 4\right) / \left(x + 1\right)$, (b) $\left(x^2 - 1\right) / \left(x - 1\right)$, (c) $|x - 1| / \left(x - 1\right)$.

1.7 / P2

Revisit after Topic 1.8. For each limit, name the evidence that determines your first method, then evaluate. (a) $\lim_{x \to 1} g\left(x\right)$ from the graph. (b) $\lim_{x \to 0} x^2 \sin\left(1/x\right)$. (c) $\lim_{x \to 0} \left(\sqrt{16 + x} - 4\right)/x$.



LESSON 02 / CED 1.8

Using the squeeze theorem

Reference topic 1.8 • LIM-1.E

1.8 / P1

Assume $\left|F\left(x\right)-7\right| \leq 3\left(x-2\right)^{2}$ near $x=2$. Give explicit lower and upper bounds for F and justify its limit.

1.8 / P2

Use the standard radian sine limit to evaluate $\lim_{x \to 0} \sin\left(5x\right)/\left(2x\right)$. Then evaluate $\lim_{x \to 0} \left(1-\cos\left(3x\right)\right)/x$.

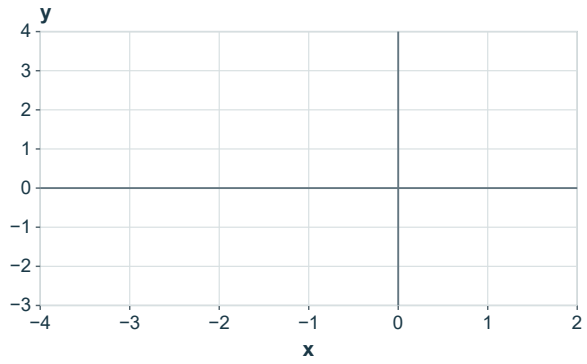
LESSON 02 / CED 1.9

Connecting representations of limits

Reference topic 1.9 • Skill 2.C

1.9 / P1

Sketch an example with $\lim_{x \to -1^-} p(x) = 2$, $\lim_{x \to -1^+} p(x) = -1$, and $p(-1) = 0$. Label the points at the break.



1.9 / P2

A plot of $q(x) = (x^2 - 25)/(x - 5)$ looks like a line through $(5, 10)$. Give table values at 4.99 and 5.01, explain the exact behavior at 5, and write a limit statement.

LESSON 02 / CED 1.10

Recognizing types of discontinuity

Reference topic 1.10 • LIM-2.A

1.10 / P1

Classify both discontinuities of $u(x) = \left(x^2 - 4\right)/[\,(x-2)\,(x+1)\,]$. Include the limiting evidence at each.

1.10 / P2

At an input a, a function has a left limit of -2, right limit 3, and point value 3. A second function has both one-sided limits equal to -2 and point value 3. Classify each and explain which can be repaired at a alone.

LESSON 02 / CED 1.11

Testing continuity at one point

Reference topic 1.11 • LIM-2.A

1.11 / P1

Let $k(x) = \left(x^2 - 9\right)/(x + 3)$ for $x \neq -3$, and $k(-3) = -6$. Give a three-part argument for or against continuity at -3.

1.11 / P2

Let $m(x) = x + 2$ for $x \leq 1$ and $m(x) = 5 - x$ for $x > 1$. A classmate notes that $m(1) = 3$ and says this proves continuity. Test the claim fully.

LESSON 02 / CED 1.12

Continuity on intervals

Reference topic 1.12 • LIM-2.B

1.12 / P1

Determine the maximal intervals of continuity for $H(x) = \ln(x+2) / \left(x^2 - 1\right)$. Show all restrictions.

1.12 / P2

For $-1 \le x \le 2$ let $p(x) = \sqrt{x+1}$ for $x \le 0$, and $p(x) = 1 + x^2$ for $x > 0$. Decide whether it is continuous on the closed interval. Check the join and both endpoints.

LESSON 02 / CED 1.13

Repairing discontinuities

Reference topic 1.13 • LIM-2.C

1.13 / P1

For $x < 3$ let $v(x) = kx + 2$, for $x > 3$ let $v(x) = x^2 - 1$, and set $v(3) = m$. Find both parameters for continuity at 3.

1.13 / P2

The function $R(x) = \left(x^2 - 1\right)/[(x - 1)(x + 2)]$ is missing two inputs. Decide at which of them it can be extended continuously. Give all possible repairs and justify each decision.

LESSON 02 / CED 1.14

Infinite limits and vertical asymptotes

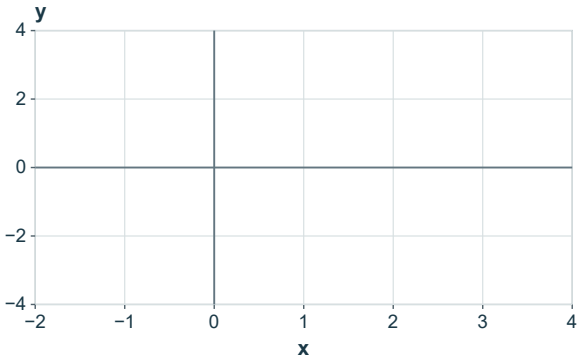
Reference topic 1.14 • LIM-2.D

1.14 / P1

Find the one-sided limits of $(2-x)/(x+3)^3$ at $x=-3$. State the vertical asymptote and whether a single two-sided infinite limit exists.

1.14 / P2

Sketch a function satisfying $\lim_{x\to 1^-} f(x) = +\infty$, $\lim_{x\to 1^+} f(x) = -\infty$, and $f(1) = 2$. Mark the asymptote and the assigned point.



LESSON 02 / CED 1.15

End behavior and horizontal asymptotes

Reference topic 1.15 • LIM-2.D

1.15 / P1

A model for average cost is $C(n) = (18n + 540)/n$ dollars per item for positive order size n. Find the end behavior and explain the role of 540 in the limit.

1.15 / P2

As $x \to +\infty$, compare $A(x) = 5x^2 + x$ with $B(x) = x^3 + 1$ using A/B. Does A itself have horizontal asymptote zero? Their rates are given by $A'(x) = 10x + 1$ and $B'(x) = 3x^2$. Compare those rates using a second ratio limit.

LESSON 02 / CED 1.16

The Intermediate Value Theorem

Reference topic 1.16 • FUN-1.A

1.16 / P1

Let $h(x) = x^3 - 4x + 1$. Use the IVT to guarantee two distinct zeros in $[0, 2]$. Identify two disjoint open intervals containing them.

1.16 / P2

A tank's volume changes continuously from 40 L at 2 min to 58 L at 8 min. A student claims it held 49 L exactly once, at 5 min. Write the conclusion justified by the data and explain both limits of that conclusion.

LESSON 02 / WITH A PARTNER

Limits and continuity

Work together on questions 1 and 2. Close the reference for question 3.

02.1 / TOGETHER

A graph approaches height 3 from the left of $x = 1$ and height -1 from the right. Its filled point is $(1, 3)$. Give the one-sided limits, the two-sided limit, and the function value.

02.2 / TOGETHER

Evaluate $\lim_{x \to 9} \left(\sqrt{x} - 3\right) / (x - 9)$ by a method that explains what happens near 9.

02.3 / SOLO

Justify $\lim_{x \to 0} x^2 \cos(1/x)$ without assigning a value to the expression at zero.

LESSON 02 / WORK INDEPENDENTLY

Limits and continuity

Show your reasoning. Finish the exit problem before checking the key.

02.4 / SOLO

For $f(x) = (2x + 1)/(x - 3)$, find the one-sided limits at 3 and the limit at positive infinity. Identify both asymptotes.

02.5 / MIXED

A table for p gives $p(-1) = 3$ and $p(2) = -4$. If p is continuous on $[-1, 2]$, justify a zero. Give a counterexample showing why the continuity assumption matters.

02.6 / EXIT

For $x \neq -2$, $h(x) = \left(x^2 - 4\right)/(x + 2)$, while $h(-2) = k$. Choose k to make h continuous at -2, and justify the choice using the limit. Classify the discontinuity if instead $k = 5$.

LESSON 03 / CED 2.1

From an average rate to an instantaneous rate

Reference topic 2.1 • CHA-2.A / CHA-2.B

2.1 / P1

For $P(t) = t^3 + 2t$, find the average rate on $[1, 1+h]$ when $h > 0$, then find the instantaneous rate at 1 using a limit. State the step that changes an average into an instantaneous rate.

2.1 / P2

A coach estimates a player's instantaneous speed from two measurements 4 s apart. What rate can the coach calculate exactly from the two values of distance traveled? What extra information would support an instantaneous estimate?

LESSON 03 / CED 2.1

From an average rate to an instantaneous rate

Reference topic 2.1 • CHA-2.A / CHA-2.B

2.1 / P3

A sensor output is $S(u) = 5/u$ for $u > 0$. Compare the average rate from $u = 2$ to $u = 3$ with the limit of average rates from 2 to u as $u \to 2$.

LESSON 03 / CED 2.2

The derivative function and the tangent line

Reference topic 2.2 • CHA-2.B / CHA-2.C

2.2 / P1

Use the limit definition to find dg/dx for $g(x) = 3/x$ on its domain. Write the tangent equation at $x = -3$.

2.2 / P2

At 6 minutes a starter contains 17 g of a substance and its quantity is increasing at 1.5 g/min at that instant. Write the two statements in function notation and construct a tangent-line model for nearby times.

LESSON 03 / CED 2.2

The derivative function and the tangent line

Reference topic 2.2 • CHA-2.B / CHA-2.C

2.2 / P3

Find $f'(x)$ from the difference quotient if $f(x) = x^2 - 2x + 1$. Use it to locate the horizontal tangent, and give that tangent's equation.

LESSON 03 / CED 2.3

Estimating a derivative from measurements or a graph

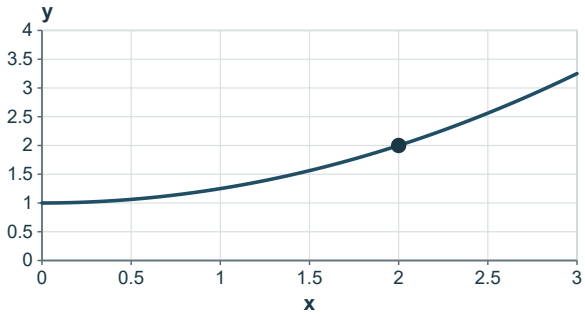
Reference topic 2.3 • CHA-2.D

2.3 / P1

Recorded total distances traveled at 7.5, 8, and 8.5 s are 33.8, 36.1, and 38.9 m. Calculate left, right, and centered estimates of the speed at 8 s. Explain your choice of a single estimate.

2.3 / P2

(a) Without a calculator, use the graph of a smooth curve through $(2,2)$. Draw a tangent there. Select two well-separated readable points on your tangent and use them to estimate the derivative. Show the selected points. (b) Calculator: a different curve is $F\left(x\right)=\sqrt{2+x^2}$. Estimate $F'\left(1\right)$ using numerical differentiation or a centered quotient with $h=0.01$. Show your calculator expression and give the estimate to three decimals.



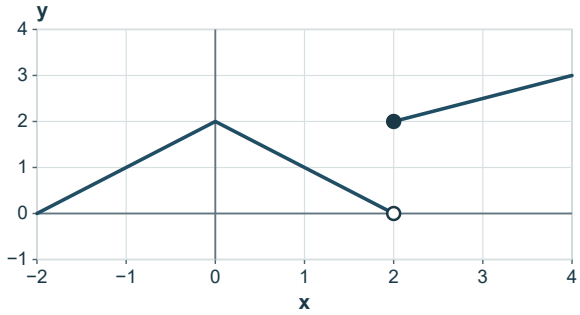
LESSON 03 / CED 2.4

Where differentiability fails

Reference topic 2.4 • FUN-2.A

2.4 / P1

On the displayed graph identify every interior input where the derivative fails. Distinguish a change in segment slope from a break in continuity.



2.4 / P2

Let $h(x) = x^2$ for $x < 0$, $h(0) = c$, and $h(x) = kx$ for $x > 0$. For which c, k is h differentiable at 0? Show both conditions.

LESSON 03 / WITH A PARTNER

From secants to derivatives

Work together on questions 1 and 2. Close the reference for question 3.

03.1 / TOGETHER

For $f(x) = x^2 - 3x$, construct the difference quotient at $x = 2$, simplify, and take its limit. Write the tangent line.

03.2 / TOGETHER

An athlete's position is 7.6, 9.1, and 10.4 m at 1.8, 2.0, and 2.2 s. Estimate the velocity at 2.0 s. Compare the one-sided average velocities.

03.3 / SOLO

Express $\lim_{x \to 4} \left(\sqrt{x} - 2\right) / (x - 4)$ as the derivative of a named function at a point, and then evaluate it.

LESSON 03 / WORK INDEPENDENTLY

From secants to derivatives

Show your reasoning. Finish the exit problem before checking the key.

03.4 / SOLO

Set $g(x) = 2x + 1$ for $x < 0$ and $g(x) = 1 - x$ for $x \geq 0$. Decide whether g is continuous and differentiable at zero. Support both decisions.

03.5 / MIXED

A graph of a differentiable function rises steadily to a smooth local maximum at $x = 2$, then falls steadily. Away from the peak its tangents are never horizontal. Describe where its derivative graph lies relative to the horizontal axis.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

03.6 / EXIT

A cylindrical tank's volume $V(t)$ satisfies $V(5) = 32$ L and $V'(5) = -1.4$ L/min, with t in minutes. Write its tangent-line model and explain the sign and units of its slope.

LESSON 04 / CED 2.5

The power rule

Reference topic 2.5 • FUN-3.A

2.5 / P1

For $g(x) = x^{4/3}$ use the power rule away from zero. Check $g'(0)$ separately with a difference quotient.

2.5 / P2

The slope of $y = x^n$ at $x = 1$ is -4, where n is a fixed exponent. Find n, then determine the slope at $x = 2$.

LESSON 04 / CED 2.6

Differentiating sums and constant multiples

Reference topic 2.6 • FUN-3.A

2.6 / P1

At $x = 3$, $f'(3) = 2$, $g'(3) = 7$. For $W(x) = 4g(x) - 3f(x) - 9$, find $W'(3)$. Do you need $f(3)$ or $g(3)$? Explain.

2.6 / P2

Choose a so that $y = 2x^3 + ax^2 - 5x + 1$ has slope 11 at $x = 2$. Show the derivative equation.

LESSON 04 / CED 2.7

Trig, exponential, and logarithmic derivatives

Reference topic 2.7 • FUN-3.A / LIM-3.A

2.7 / P1

An angle is measured in radians. For $p(x) = 2e^x + 3\cos x - \sin x$, find the tangent equation at zero.

2.7 / P2

Identify the function and base point in $\lim_{h \to 0} \left(e^3 - e^{3+h}\right)/h$. Evaluate the limit, explaining the sign.

LESSON 04 / CED 2.7

Trig, exponential, and logarithmic derivatives

Reference topic 2.7 • FUN-3.A / LIM-3.A

2.7 / P3

Suppose $W(x) = e^x - 2\cos x$. Evaluate $\lim_{h \to 0} [W(2+3h) - W(2)]/h$ by recognizing a derivative. Account for the factor multiplying it.

LESSON 04 / CED 2.8

The product rule

Reference topic 2.8 • FUN-3.B

2.8 / P1

A store holds $n(t)$ kilograms of bulk coffee, currently valued at $p(t)$ dollars per kilogram. At $t = 2$ hours, $n = 40$, $p = 7$, $n' = 3$ kg/hour, and $p' = -0.25$ dollars per kilogram per hour. Find the rate of change of the inventory value np and interpret its sign.

2.8 / P2

Differentiate $(x - 2)\left(x^2 + x + 1\right)$ in two ways and show the answers agree.

LESSON 04 / CED 2.9

The quotient rule

Reference topic 2.9 • FUN-3.B

2.9 / P1

Find the tangent equation to $h(x) = x/(1 + x^2)$ at $x = 1$. Explain why the denominator of h' is nonzero for real inputs.

2.9 / P2

A student gives u'/v' as the derivative of u/v. Use $u(x) = x$ and $v(x) = 2x$ to test the claim on $x > 0$, then verify using the quotient rule.

LESSON 04 / CED 2.10

Derivatives of the four remaining trig functions

Reference topic 2.10 • FUN-3.B

2.10 / P1

Start from $\csc x = 1/\sin x$. Derive its derivative and use it to find the slope at $x = 5\pi/6$.

2.10 / P2

Find the derivative of $g(x) = \tan x - 2\cot x + \sec x$. Give its domain and evaluate $g'(\pi/4)$.

LESSON 04 / WITH A PARTNER

Choosing a differentiation rule

Work together on questions 1 and 2. Close the reference for question 3.

04.1 / TOGETHER

For $x > 0$, rewrite $F(x) = 3\sqrt{x} - 4/x^2 + 7$ using exponents. Find $F'(x)$ and $F'(4)$.

04.2 / TOGETHER

Differentiate $p(x) = (x + 2)\,e^x$. Evaluate at $x = 0$ and give a quick reason the sign makes sense there.

04.3 / SOLO

Find $f'(x)$ for $f(x) = \sin x/(x + 2)$ and evaluate at 0.

LESSON 04 / WORK INDEPENDENTLY

Choosing a differentiation rule

Show your reasoning. Finish the exit problem before checking the key.

04.4 / SOLO

A model is $H(x) = 3^x + \ln x - \cot x$, where $x > 0$ and $\sin x \neq 0$. Find $H'(x)$ and identify the origin of the sign on the last term.

04.5 / MIXED

Use the values at $x = 3$ to find $(uv)'(3)$ and $(u/v)'(3)$. Show the substitutions.

Input	u	v	u'	v'
$x = 3$	2	-4	5	1

04.6 / EXIT

A particle has position $s(t) = t^2 \sin t$ meters for $t \geq 0$ seconds, with the sine evaluated in radians. Find $s'(t)$ and the velocity at $t = \pi$. Interpret its sign.

LESSON 05 / CED 3.1

Rates through composed functions

Reference topic 3.1 • FUN-3.C

3.1 / P1

Find the tangent equation for $y = \ln\left(1 + (2x - 1)^2\right)$ at $x = 1$. Show where each numerical factor comes from.

3.1 / P2

Let T convert altitude to temperature. At 2 h a climber is at 1.8 km, climbing at 0.4 km/h. At that height $T' = -6$ °C/km. Calculate the local temperature rate and label units.

LESSON 05 / CED 3.2

Differentiating an implicit relation

Reference topic 3.2 • FUN-3.D

3.2 / P1

A student claims that $xy + \sin y = 2$ passes through $(2, \pi/2)$. Test the proposed point and find the correct x-coordinate when $y = \pi/2$. Find the slope there.

3.2 / P2

The curve $y^2 + xy = 12$ passes through $(1, 3)$. Determine its tangent equation, with enough work to show both occurrences of y' in the differentiated equation.

LESSON 05 / CED 3.3

Slope of an inverse function

Reference topic 3.3 • FUN-3.E

3.3 / P1

Let g be the inverse of $f(x) = e^x + x$ on the real line. Find the tangent to g at input 1.

3.3 / P2

A strictly decreasing F satisfies $F(-2) = 5$, $F(5) = -1$, $F'(-2) = -3$, and $F'(5) = -6$. A student says $\left(F^{-1}\right)'(5) = -1/6$. Identify the incorrect input and correct the result.

LESSON 05 / CED 3.4

Differentiating inverse trigonometric functions

Reference topic 3.4 • FUN-3.E

3.4 / P1

For $q(x) = \arcsin\left((x-2)/3\right)$, find $q'(x)$, the interval where that derivative formula applies, and $q'(2)$.

3.4 / P2

Let $y = \arctan x$. Use the equation $\tan y = x$, together with a trig identity, to derive dy/dx. Explain why the resulting expression has no real singularity.

LESSON 05 / CED 3.5

Choosing and combining derivative rules

Reference topic 3.5 • Skill 1.C

3.5 / P1

For $F(x) = \ln\left(1 + x^2\right) / (x + 1)$, identify the first rule and the nested rule, then find $F'(0)$.

3.5 / P2

For each task, state the feature that determines your first rule, then calculate. (a) Differentiate $\sin\left(x^2\right)$. (b) Find the slope of $x^2 + xy = 6$ at $(2, 1)$. (c) The table gives data for a differentiable invertible function f. Find $\left(f^{-1}\right)'(3)$.

x	$f(x)$	$f'(x)$
0	1	1
1	3	2
2	6	4

LESSON 05 / CED 3.6

Second and higher derivatives

Reference topic 3.6 • FUN-3.F

3.6 / P1

A sensor records $H'(1.9) = -4.6$ and $H'(2.1) = -4.0$ liters per minute. Estimate $H''(2)$ and state what its sign says about the outflow if H is the amount in the tank and no water enters.

3.6 / P2

Let $y = p(x) = x^4 - 2x^3 + x$. Find $p^{(3)}(-1)$ and d^4y/dx^4. Verify that the fourth derivative is constant.

LESSON 05 / WITH A PARTNER

Chain, implicit, and inverse rules

Work together on questions 1 and 2. Close the reference for question 3.

05.1 / TOGETHER

For $y = [1 + \sin(3x)]^4$, name the layers, then find y' and its value at zero.

05.2 / TOGETHER

For $x^2 + xy + y^2 = 7$, find dy/dx and evaluate it at $(1, 2)$. Identify both terms that come from differentiating xy.

05.3 / SOLO

A differentiable, strictly increasing function has $f(2) = 5$, $f'(2) = 4$, and $f'(5) = 7$. Use the data to find $\left(f^{-1}\right)'(5)$ and the equation of the inverse's tangent line at input 5. Assume its inverse is differentiable there.

LESSON 05 / WORK INDEPENDENTLY

Chain, implicit, and inverse rules

Show your reasoning. Finish the exit problem before checking the key.

05.4 / SOLO

Find the derivatives of $p(x) = \arcsin(2x)$ and $q(x) = \arctan(x^2)$, and give the domain on which each derivative formula applies.

05.5 / MIXED

A machine processes a signal in two steps. First $u = g(t)$, then $y = f(u)$. At $t = 3$, $g(3) = 2$ and $g'(3) = -4$. A table gives $f'(2) = 5$ and $f'(3) = -1$. Find dy/dt at 3 and explain the sign.

05.6 / EXIT

On the upper semicircle $x^2 + y^2 = 25$, find the second derivative at $(3, 4)$. Start from the first implicit-derivative equation.

LESSON 06 / CED 4.1

Interpreting a derivative in context

Reference topic 4.1 • CHA-3.A

4.1 / P1

A sheet's temperature $T(x)$ is in degrees Celsius at x centimeters from its left edge. At $x = 6$, $T = 54$ and $T' = -1.2$. Interpret both. Do they determine the exact temperature at $x = 7$?

4.1 / P2

A player has covered $D(t)$ meters after t seconds, with $D(0) = 0$. At 12 s she has covered 72 m and her speed is 4.5 m/s. Write both statements and explain the difference between $D'(12)$ and $72/12$.

LESSON 06 / CED 4.2

Position, velocity, acceleration, and speed

Reference topic 4.2 • CHA-3.B

4.2 / P1

For $v(t) = t(t-4)$ m/s on $0 < t < 5$ seconds, mark where v and a change sign. Find the intervals where speed increases and where motion is in the negative direction.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

4.2 / P2

With east positive and time in seconds, a cart has $s(t) = 8 + 6t - t^2$ meters on $[0, 5]$. When does it stop? Compare direction, speed, and acceleration at 2 and 4 s.

LESSON 06 / CED 4.3

Rates beyond straight-line motion

Reference topic 4.3 • CHA-3.C

4.3 / P1

Reservoir depth is $h(t) = 6 + 0.6t - 0.05t^2$ meters for $0 \le t \le 9$ hours. Find the period of rising depth and interpret $h'(8)$.

4.3 / P2

A factory produces at $R(t) = 8 + 2t/(1 + t)$ crates per hour for $t \ge 0$ hours. What are its production rate at $t = 3$ and the rate at which production per hour is changing?

LESSON 06 / CED 4.4

Build a related-rates equation

Reference topic 4.4 • CHA-3.D

4.4 / P1

Two boats move on perpendicular straight channels away from their crossing. Draw their distances x, y from the crossing and separation d. Derive an equation for d' using their signed rates.

4.4 / P2

A box has a square base of side $a(t)$ and height $h(t)$. Write its volume-rate equation. At positive dimensions, what condition keeps volume constant at that instant?

LESSON 06 / CED 4.5

Solve related-rates problems

Reference topic 4.5 • CHA-3.E

4.5 / P1

A 2-m-tall person walks away from a 6-m light pole at 1.8 m/s on level ground. Draw similar triangles and find the speed of the shadow tip relative to the pole.

4.5 / P2

A trough is 5 m long. Its vertical end is a triangle, tip down, 2 m wide at its 1-m height. Water drains at 0.20 m^3/min. Find the depth rate when depth is 0.5 m.

LESSON 06 / CED 4.6

Local linearity and tangent estimates

Reference topic 4.6 • CHA-3.F

4.6 / P1

Use the tangent at $x = 0$ to estimate $e^{-0.08}$. Give the predicted change and explain the error direction.

4.6 / P2

A rope under load F newtons has length $\ell(F)$ cm. Suppose $\ell(40) = 106$, $\ell'(40) = 0.18$ cm/N, and the slope decreases on $[38, 40]$. Estimate $\ell(38.5)$ and judge the error.

LESSON 06 / CED 4.7

Limits of indeterminate quotients

Reference topic 4.7 • LIM-4.A

4.7 / P1

Evaluate $\lim_{x \to 0} (1 - \cos(3x))/x^2$. If using the rule twice, justify each use.

4.7 / P2

Suppose $f(2) = 5$, $g(2) = 8$, $f'(2) = -3$, $g'(2) = 2$, and the derivatives are continuous near 2. Compare the limits of $(f(x) - 5)/(g(x) - 8)$ and $f(x)/g(x)$ as $x \to 2$.

LESSON 06 / WITH A PARTNER

Rates, local estimates, and limits

Work together on questions 1 and 2. Close the reference for question 3.

06.1 / TOGETHER

At $t = 4$ s a runner has velocity -3 m/s and acceleration 2 m/s² on a straight track with a marked positive direction. State her direction, speed, and whether her speed is increasing or decreasing at that instant.

Quantity	Response
Direction	
Speed	
Speed increasing or decreasing?	
Reason using the signs	

06.2 / TOGETHER

A circular patch expands with $dr/dt = 0.8$ cm/min. Find its area-growth rate at $r = 5$ cm. Why would setting $r = 5$ before differentiating fail?

06.3 / SOLO

Use the tangent to $f(x) = \sqrt{x}$ at 25 to approximate $\sqrt{24.4}$. Decide whether the estimate is high or low using the sign of f''.

LESSON 06 / WORK INDEPENDENTLY

Rates, local estimates, and limits

Show your reasoning. Finish the exit problem before checking the key.

06.4 / SOLO

Evaluate $\lim_{x \to 0} \ln(1+3x) / \sin(2x)$. Check the form before selecting a method.

06.5 / MIXED

On a straight track, $v(t) = t^2 - 4t + 3$ m/s for $0 \le t \le 4$ s. Use sign changes to find when direction reverses. Is the particle speeding up at $t = 2.5$ s?

Mark the inputs where a sign can change. Label each row with the expression you are testing.

06.6 / EXIT

A sensor table gives $T(8) = 21.6$ °C and $T'(8) = -0.3$ °C/min. Estimate $T(8.4)$ and explain why the table cannot tell you whether your estimate is high or low.

LESSON 07 / CED 5.1

A secant slope that a tangent must match

Reference topic 5.1 • FUN-1.B

5.1 / P1

For $h(x) = 1/x$ on $[1, 4]$, verify the hypotheses, calculate the secant slope, and find all matching inputs.

5.1 / P2

A cart has position 3 m at $t = 1$ s and 17 m at $t = 5$ s. Its position is continuous on $[1, 5]$ and differentiable inside. A technician claims its speed never exceeded 3 m/s. Evaluate the claim.

LESSON 07 / CED 5.2

Absolute extrema, local extrema, and critical numbers

Reference topic 5.2 • FUN-1.C

5.2 / P1

Let $p(x) = 1 - (x-2)^2$ on $(0,4)$. Determine which absolute extrema exist. Does the EVT guarantee your answer?

5.2 / P2

On $[-1,3]$, let $f(x) = x$ except that $f(1) = 10$. Does the EVT apply? Determine the extreme values directly.

LESSON 07 / CED 5.3

Intervals of increase and decrease

Reference topic 5.3 • FUN-4.A

5.3 / P1

For $f(x) = x + 4/x$, use a sign chart to identify all intervals of increase and decrease. Mark the excluded input.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

5.3 / P2

If $q'(x) = (2 - x)e^x$, compare $q(3)$ with $q(6)$. Does the same argument compare $q(1)$ and $q(4)$?

LESSON 07 / CED 5.4

The first derivative test

Reference topic 5.4 • FUN-4.A

5.4 / P1

Given $f'(x) = (x+1)(x-2)(x-5)$ on the real line, classify every critical number.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

5.4 / P2

For $g(x) = x^{2/3}(x-5)$, classify both critical numbers using the first derivative test.

LESSON 07 / CED 5.5

Comparing candidates for absolute extrema

Reference topic 5.5 • FUN-4.A

5.5 / P1

Fill the candidate table for $h(x) = x^4 - 8x^2$ on $[-3, 1]$. Report the absolute extrema.

Candidate input	Reason	Function value

5.5 / P2

Find the absolute extreme values of $q(x) = \sin x + \cos x$ on $[0, \pi]$. Explain why both extreme values must be attained and how you know the candidate list is complete.

LESSON 07 / CED 5.6

Concavity and changes of concavity

Reference topic 5.6 • FUN-4.A

5.6 / P1

For $h(x) = \sqrt[3]{x}$, decide whether the origin is an inflection point. Include the intervals of concavity and address the undefined derivative there.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

5.6 / P2

A function is twice differentiable on the real line and $f''(x) = (x+2)^2(x-3)$. Find all inflection inputs and justify rejecting any other candidate.

LESSON 07 / CED 5.7

The second derivative test

Reference topic 5.7 • FUN-4.A

5.7 / P1

For $p(x) = xe^{-x}$, find and classify the stationary point by computing the first and second derivatives. Is the result also a global extremum on the real line?

5.7 / P2

A student says $g(x) = -(x-3)^6$ cannot have an extremum at 3 because $g''(3) = 0$. Evaluate the claim and classify the point.

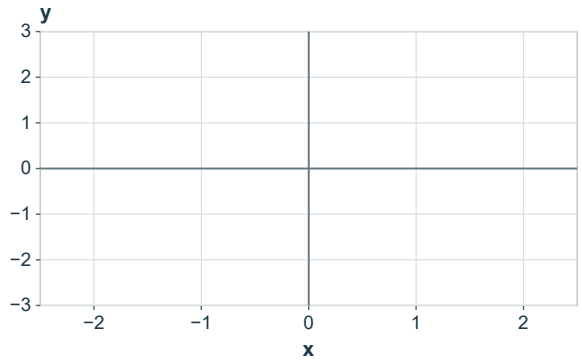
LESSON 07 / CED 5.8

Sketching from slopes and curvature

Reference topic 5.8 • FUN-4.A

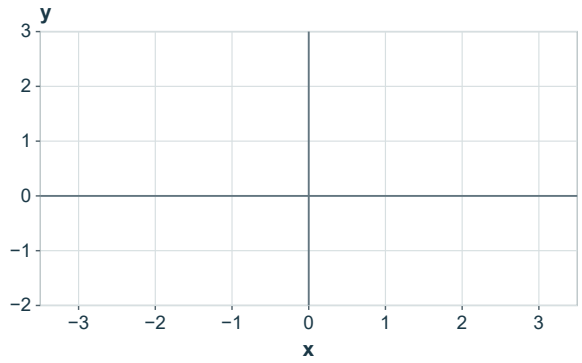
5.8 / P1

On $-2 \le x \le 2$, a smooth f has $f' = 3x^2 - 3$ and $f(0) = 0$. Sketch f; mark both local turns and any inflection point.



5.8 / P2

A graph is made of segments joining $(-3,1)$, $(-1,-1)$, $(1,3)$, $(3,3)$ in order. Sketch its derivative on $(-3,3)$, including what happens at the two interior joins.



LESSON 07 / CED 5.9

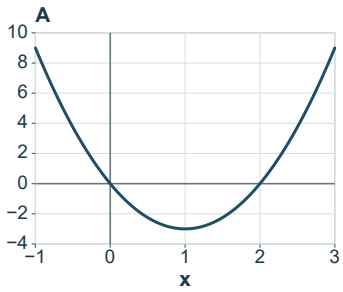
Connecting three graphs

Reference topic 5.9 • FUN-4.A

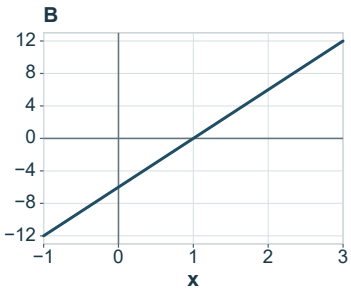
5.9 / P1

The three graphs are a function f, its first derivative, and its second derivative, in an unknown order. Assign each role to A, B, or C. Justify your assignment using a shared zero and horizontal tangent, then an interval of increase or decrease. Locate the local extrema and inflection point of f.

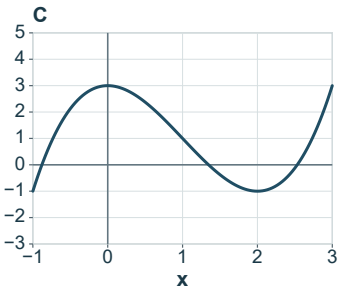
Graph A



Graph B



Graph C



5.9 / P2

A depth function D has $D'(t) > 0$ from 0 to 8 minutes. The graph of D' rises until minute 5 and then falls. Describe D on both intervals. Does the depth peak at minute 5?

LESSON 07 / WITH A PARTNER

Reading a function through its derivatives

Work together on questions 1 and 2. Close the reference for question 3.

07.1 / TOGETHER

A smooth function has $f' > 0$ on $(0, 2)$, $f' < 0$ on $(2, 5)$, and $f' > 0$ on $(5, 7)$. Classify the interior turns and justify each.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

07.2 / TOGETHER

Let $p(x) = x^3 - 6x^2$ on $[0, 5]$. Find the intervals of concavity and the coordinates of each inflection point.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

07.3 / SOLO

Find the absolute extrema of $h(x) = x^4 - 2x^2$ on $[-2, 1]$. List every location attaining each.

LESSON 07 / WORK INDEPENDENTLY

Reading a function through its derivatives

Show your reasoning. Finish the exit problem before checking the key.

07.4 / SOLO

A runner's distance $d(t)$ is continuous from minute 1 to minute 5 and differentiable between those times. Distances are 310 m and 1110 m at the endpoints. What instantaneous speed must she reach? Justify.

07.5 / MIXED

For $r(x) = |x|$ on $[-2, 2]$, the two endpoint values agree. Does the Mean Value Theorem guarantee a horizontal tangent between them? Describe the absolute minimum.

07.6 / EXIT

Throughout $(1, 4)$, the graph of f' is below the horizontal axis and rises as x increases. Is $f(4)$ larger or smaller than $f(1)$ if f is continuous on $[1, 4]$? Describe its concavity inside the interval.

LESSON 08 / CED 5.10

Building an optimization model

Reference topic 5.10 • FUN-4.B

5.10 / P1

Two nonnegative numbers x, y satisfy $x + 2y = 24$. Model and find the greatest product xy. Give both numbers and the product.

5.10 / P2

A rectangle in the first quadrant has one corner at the origin and its opposite corner on $y = 12 - 3x$, with sides parallel to the axes. Sketch it, identify the feasible domain, and maximize its area.

LESSON 08 / CED 5.11

Interpreting optimal values in context

Reference topic 5.11 • FUN-4.C

5.11 / P1

A shuttle operator's model for daily profit is $P(n) = 100n - 4n^2 - 60$ dollars for a whole number n of trips from 8 to 15. Identify every most profitable schedule and the profit.

5.11 / P2

A pump's operating temperature follows $T(t) = 18 + (t - 7)^2/2$ degrees Celsius for $0 \le t \le 5$ hours. A report gives 18 degrees as the minimum measured during the shift. Correct the report using the time and temperature.

LESSON 08 / CED 5.12

Critical points and curvature of implicit curves

Reference topic 5.12 • FUN-4.D / FUN-4.E

5.12 / P1

For $y^2 = x^3 - 3x + 3$, find all horizontal tangent points. Use the second derivative to classify each as a local maximum or minimum of its branch.

5.12 / P2

For $x = y^3 - 3y$, find every point with a vertical tangent and decide whether a horizontal tangent is possible. On which parts does y decrease as x increases? At the point $\left(0, \sqrt{3}\right)$, also calculate y'' and state the branch's concavity.

LESSON 08 / WITH A PARTNER

Constraints, optimal values, and curves

Work together on questions 1 and 2. Close the reference for question 3.

08.1 / TOGETHER

A rectangle has perimeter 36 cm. Let w be its width. Write its area in terms of w, state the feasible interval, and find the largest possible area.

08.2 / TOGETHER

On $x^2 + xy + 2y^2 = 8$, locate both points with $x = 0$. Find the tangent line at each point and determine whether the lines are parallel.

08.3 / SOLO

A company models the cost of a production run by $C(q) = q^2 - 24q + 190$ dollars for $5 \le q \le 18$, where q is measured in hundreds of items. Find the minimum cost and the number of items.

LESSON 08 / WORK INDEPENDENTLY

Constraints, optimal values, and curves

Show your reasoning. Finish the exit problem before checking the key.

08.4 / SOLO

For $4x^2 + y^2 = 16$, identify the coordinates of every horizontal and vertical tangent point.

08.5 / MIXED

A 12 cm square sheet becomes an open box when squares of side x are cut from its corners and the sides are folded up. Write its volume. Explain why $x = 6$ cannot maximize the volume even though it makes the derivative zero.

08.6 / EXIT

A rectangle lies above the horizontal axis with its bottom corners on that axis and its top corners on $y = 12 - x^2$, symmetric about the vertical axis. Choose its width and height to maximize its area. Justify over the feasible interval.

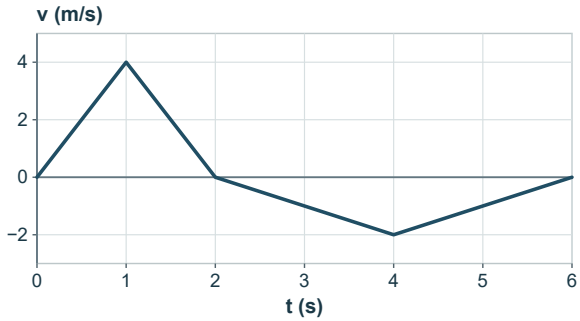
LESSON 09 / CED 6.1

Signed area and accumulated change

Reference topic 6.1 • CHA-4.A

6.1 / P1

The graph shows vertical velocity in m/s. The elevator begins at height 12 m. Determine its final height and total vertical distance. Write one sentence explaining why the results differ.



6.1 / P2

The temperature in a cooler starts at 7°C. Over the next 3 hours its rate of change falls linearly from 4 to 0 degrees Celsius per hour. Over the following 2 hours the rate is -2 throughout. What is its final temperature, and when is it warmest?

LESSON 09 / CED 6.2

Choosing and interpreting an approximation

Reference topic 6.2 • LIM-5.A

6.2 / P1

The increasing rate of ticket sales is measured in tickets per hour at 0, 1, 3, and 6 hours. The four values are 8, 12, 18, and 24. Fill each row with the estimated contribution in tickets. Give the three totals and identify the estimates guaranteed to bound sales.

Interval (h)	Left	Right	Trapezoid
0 to 1			
1 to 3			
3 to 6			

6.2 / P2

Use a calculator and three equal midpoint rectangles to estimate $\int_0^3 e^{-x}\,dx$. Show the inputs and round only the final result to three decimals. Justify its error direction.

LESSON 09 / CED 6.3

Between sums and integrals

Reference topic 6.3 • LIM-5.B / LIM-5.C

6.3 / P1

A pollution sensor samples a continuous deposition rate $r(t)$ in grams per minute on $[3, 9]$. Use right endpoints to write a finite n-term estimate and its limiting integral. Explain the units of one term and of the limit.

6.3 / P2

For $\lim_{n \to \infty} \sum_{i=1}^{n} (4/n) \cos(-1 + (4i - 2)/n)$, identify the sampling rule, write the matching integral, and explain why a right sum on the same interval would have the same limit.

LESSON 09 / CED 6.6

Computing with the properties of integrals

Reference topic 6.6 • FUN-6.A

6.6 / P1

A curve from $x = -2$ to $x = 2$ is the lower semicircle of radius 2 centered at the origin. From $x = 2$ to $x = 5$, a line joins $(2, 0)$ to $(5, 4)$. Find $\int_{5}^{-2} f(x)\,dx$ and $\int_{-2}^{5} |f(x)|\,dx$.

6.6 / P2

A bounded function equals $x + 1$ for $0 \le x < 2$, is assigned the value 9 at 2, and equals -1 for $2 < x \le 5$. Calculate $\int_{0}^{5} [3f(x) + 2]\,dx$. Would choosing $f(2) = -100$ change the result?

LESSON 09 / WITH A PARTNER

From rate samples to a definite integral

Work together on questions 1 and 2. Close the reference for question 3.

09.1 / TOGETHER

A rate is 3, 5, and 4 L/min at minutes 0, 2, and 5. Compute left and right sums for total inflow on $[0, 5]$.

09.2 / TOGETHER

Velocity is linear between consecutive sample times in the table. Compute the displacement and total distance.

Time (s)	0	2	5
Velocity (m/s)	4	0	-2

09.3 / SOLO

Express $\int_1^3 \sqrt{1+x}\,dx$ as the limit of a midpoint sum over n equal intervals.

LESSON 09 / WORK INDEPENDENTLY

From rate samples to a definite integral

Show your reasoning. Finish the exit problem before checking the key.

09.4 / SOLO

Rewrite $\lim_{n \to \infty} \sum_{i=1}^{n} (4/n) \cos(1 + 4i/n)$ as a definite integral, identifying the interval and sample location.

09.5 / MIXED

Given $\int_0^2 f(x)\,dx = 5$ and $\int_2^7 f(x)\,dx = -3$, find $\int_7^0 (2f(x) + 1)\,dx$.

09.6 / EXIT

A smooth valve flow rate, in L/min, increases and is concave down on minutes 0 through 4. Rates at minutes 0, 1, 4 are 2, 4, 7. Give a trapezoidal estimate and say whether increasing or concave down justifies its error direction.

LESSON 10 / CED 6.4

A new function from a moving endpoint

Reference topic 6.4 • FUN-5.A

6.4 / P1

A hose runs for $g(x) = x^2 + 1$ minutes when its control reads $x > 0$. Its flow after t minutes is $r(t) = 2 + \sin(t^2)$ L/min. Write an integral for the delivered volume and determine how fast the volume changes per unit of the control reading at $x = 1$.

6.4 / P2

A continuous function f has $f(2) = 5$ and $f(6) = -1$. Define $M(x) = \int_x^{3x} f(t)\,dt$. Find $M'(2)$. Explain why these same two function values do not determine $M(2)$.

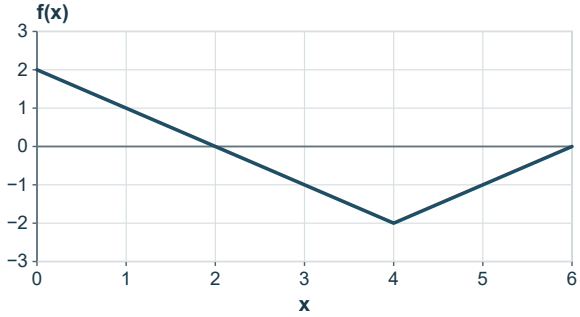
LESSON 10 / CED 6.5

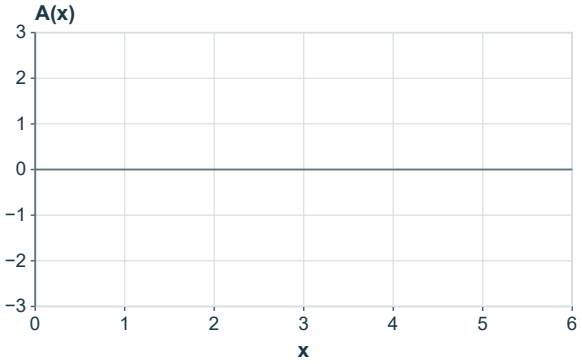
Reading an accumulation graph from its rate

Reference topic 6.5 • FUN-5.A

6.5 / P1

The plotted graph is f. Sketch $A(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 6$. Label the values at $x = 0, 2, 4, 6$ and indicate concavity on both sides of 4.





6.5 / P2

The differentiable function f has zeros only at 1 and 3 in $(0,5)$. Its signs on the successive intervals are positive, negative, positive. Its signed areas there are 4, -7, and 5. For $G(x) = 2 + \int_0^x f(t)\,dt$, locate its absolute extrema on $[0,5]$ and justify.

LESSON 10 / CED 6.7

Evaluating integrals with the fundamental theorem

Reference topic 6.7 • FUN-6.B

6.7 / P1

A particle's velocity is $v(t) = 3t^2 - 6t$ m/s on $[0, 3]$. Compute its displacement and the distance traveled, using the FTC on each interval required.

6.7 / P2

For positive x, someone proposes $F(x) = x \ln x - x$ for $f(x) = \ln x$. Verify and then evaluate $\int_1^{e^2} \ln x \, dx$. State which hypothesis licenses endpoint subtraction.

LESSON 10 / WITH A PARTNER

Accumulation and the fundamental theorem

Work together on questions 1 and 2. Close the reference for question 3.

10.1 / TOGETHER

Velocity is $v(t) = 3t^2 - 4$ m/s from second 0 to second 2. A particle starts at position 5 m. Use an antiderivative to find its displacement and final position.

10.2 / TOGETHER

A continuous rate r has signed areas 5 on $[0, 2]$ and -8 on $[2, 5]$. If $B(x) = 4 + \int_0^x r(t)\,dt$, find $B(2)$ and $B(5)$. Can those values alone establish that B decreases everywhere on $(2, 5)$?

10.3 / SOLO

Differentiate $J(x) = \int_{x^2}^{3} \sqrt{1 + t^2}\,dt$ and explain the sign of each factor.

LESSON 10 / WORK INDEPENDENTLY

Accumulation and the fundamental theorem

Show your reasoning. Finish the exit problem before checking the key.

10.4 / SOLO

The graph of r connects $(0,-2)$, $(2,0)$, $(4,2)$ by straight lines. For $A(x) = \int_0^x r(t)\,dt$, give the minimum value on $[0,4]$, its location, and the concavity of A.

10.5 / MIXED

For $x > 0$, differentiate $K(x) = \int_x^{x^2} e^t\,dt$. Evaluate $K'(1)$ and explain why $K(1) = 0$ permits a nonzero derivative.

10.6 / EXIT

A tank contains 30 L at minute 1. Its net flow is $r(t) = 6 - 2t$ L/min on $[1,5]$. When is the amount greatest, and what is that amount? Justify with the rate and an integral.

LESSON 11 / CED 6.8

Families of antiderivatives

Reference topic 6.8 • FUN-6.C

6.8 / P1

A curve has slope $f'\left(x\right) = 2x + 1 / \sqrt{1 - x^2}$ for $-1 < x < 1$ and passes through $(0, 6)$. Find its equation on that interval. Verify the derivative, initial value, and domain.

6.8 / P2

Construct the unique function H with $H(1) = 7$ and $H'(x) = \cos\left(x^2\right) + 2x$. Express the part that has an elementary antiderivative as a polynomial, and explain how to verify your result.

LESSON 11 / CED 6.9

Undoing a chain rule with substitution

Reference topic 6.9 • FUN-6.D

6.9 / P1

On $x > 0$, find $\int (\ln x)^3/x\,dx$. Verify the answer and explain what happens to the $1/x$ during substitution.

6.9 / P2

A sensor records net flow $r(t) = t/(4 + t^2)$ L/min from minute 0 to minute 2. Find the exact volume delivered using a new variable and show its two bounds.

LESSON 11 / CED 6.10

Rewriting before integrating

Reference topic 6.10 • FUN-6.D

6.10 / P1

Evaluate $\int_1^3 \left(x^2+4x+5\right)/\left(x+1\right)\,dx$ by division. Show the decomposition and endpoint calculation.

6.10 / P2

Find $\int dx/\left(x^2-6x+10\right)$. A classmate proposes $\ln\left(x^2-6x+10\right)$ instead. Verify your answer and identify the extra factor in the proposal.

LESSON 11 / CED 6.14

Selecting an integration method

Reference topic 6.14 • Skill 1.C

6.14 / P1

Write the first move and a complete answer for each: (a) $\int_1^2 2x/\left(1+x^2\right)\,dx$, (b) $\int_1^2 dx/\left(1+x^2\right)$, (c) $\int_1^2 x^2/\left(1+x^2\right)\,dx$. Explain what changes when only the numerator changes.

6.14 / P2

A pump gives flow $r\left(t\right)=\left(2t+3\right)/\left(t^2+2t+5\right)$ L/min on $[1,3]$. Select a method, find the exact amount delivered, and check its size using the minimum and maximum flow rates on the interval. A calculator may be used for a final decimal check.

LESSON 11 / WITH A PARTNER

Choosing an antiderivative

Work together on questions 1 and 2. Close the reference for question 3.

11.1 / TOGETHER

A cart has velocity $v(t) = 4t - 3$ m/s and position $s(1) = 6$ m. Find its position function and verify both conditions.

11.2 / TOGETHER

Find $\int \frac{6x}{3x^2 + 4}\,dx$. Explain how you know the logarithm is defined for all real x.

11.3 / SOLO

Evaluate $\int_0^{\pi/2} \sin x\, e^{\cos x}\,dx$ by substitution. Show the new bounds.

LESSON 11 / WORK INDEPENDENTLY

Choosing an antiderivative

Show your reasoning. Finish the exit problem before checking the key.

11.4 / SOLO

Find $\int \left(x^2+1\right)/\left(x+1\right)\,dx$ for $x > -1$. Verify by putting the derivative over a common denominator.

11.5 / MIXED

A student proposes $\ln\left(x^2+4x+5\right)$ as an antiderivative for $1/\left(x^2+4x+5\right)$. Check the proposal and give a correct antiderivative.

11.6 / EXIT

A sensor reports net water flow $r\left(t\right) = 2t/\left(1+t^2\right)$ L/min from minute 0 through minute 3. The tank has 7 L at minute 0. Find the quantity at minute 3 and explain why its units are liters.

LESSON 12 / CED 7.1

Translate a rate description into an equation

Reference topic 7.1 • FUN-7.A

7.1 / P1

A cooler initially at 12 degrees Celsius warms in air at 24 degrees. Its warming rate is proportional to the difference between air and cooler temperatures. Define a positive constant and write the initial-value problem. Why would the same rule cool a hotter cooler?

7.1 / P2

A bin receives grain at $12 + 2t$ kg/min on $0 \le t \le 4$ minutes and loses grain at 15 kg/min. It starts with 60 kg. Model the mass and determine when it begins to rise.

LESSON 12 / CED 7.2

Verify a proposed solution

Reference topic 7.2 • FUN-7.B

7.2 / P1

On $x > 0$, test $y = x \ln x$ for $xy' = x + y$. Can it also satisfy $y(1) = 1$?

7.2 / P2

Test $y = 2 + 3e^{-x}$ and $y = 2 + 3e^{x}$ for $y' = 2 - y$, $y(0) = 5$. Compare the point test and derivative test.

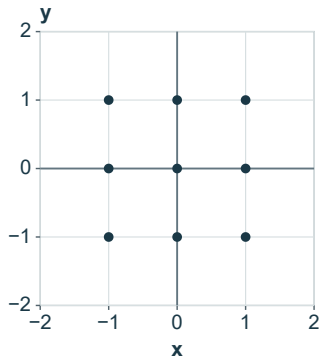
LESSON 12 / CED 7.3

Construct and identify slope fields

Reference topic 7.3 • FUN-7.C

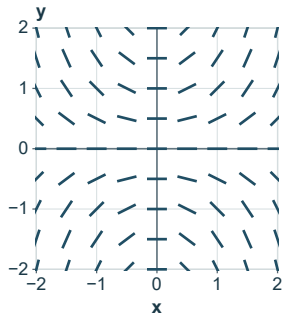
7.3 / P1

Draw a nine-segment field for $y' = y - 2x$ at every point with coordinates selected from $-1, 0, 1$. Center a segment on each point and identify the horizontal segments.



7.3 / P2

Match the displayed field to $y' = xy$, $y' = x + y$, or $y' = y - x$. Use a horizontal segment away from the origin and a nonhorizontal one.



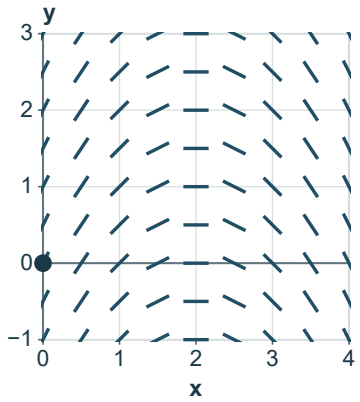
LESSON 12 / CED 7.4

Reason about a solution using a field

Reference topic 7.4 • FUN-7.C

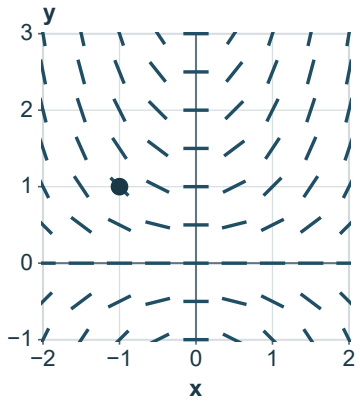
7.4 / P1

On the field $y' = 2 - x$, sketch the solution through $(0, 0)$ for $0 \le x \le 4$. Estimate the input of its highest point. How does the curve through $(0, -1)$ differ?



7.4 / P2

On the field $y' = xy$, sketch the solution through $(-1, 1)$ for $-1 \le x \le 1$. Where does it turn? Compare it with the solution through $(-1, 0)$.



LESSON 12 / CED 7.6

Separate variables for a general solution

Reference topic 7.6 • FUN-7.D

7.6 / P1

Find the general solution of $y' = (2x + 1)\,y^2$, including all constant solutions and any restrictions on a nonconstant solution.

7.6 / P2

Find an implicit general solution of $y' = e^x / \left(1 + y^2\right)$. Could any constant function solve it?

LESSON 12 / CED 7.7

Choose the solution through an initial point

Reference topic 7.7 • FUN-7.E

7.7 / P1

Solve $y' = y^2$ with $y(1) = -2$ and state the maximal interval through 1. Verify both conditions.

7.7 / P2

If $y' = 3x^2/(2y)$ and $y(2) = 3$, find $y(1)$ exactly and give the maximal interval through 2.

LESSON 12 / CED 7.8

Exponential growth and decay models

Reference topic 7.8 • FUN-7.F / FUN-7.G

7.8 / P1

A process doubles its output rate every five hours. Assume $R'(t)$ is proportional to $R(t)$, and $R(0) = 16$ L/h. Determine the coefficient and differential equation. Give its general solution family, choose the particular model, and find $R'(10)$ with units.

7.8 / P2

A sample falls from 90 g to 72 g in four days at a rate proportional to its mass. Give the general family for the fitted differential equation, then its particular mass function. When does it reach 45 g? Why would constant grams-per-day loss give a different answer?

LESSON 12 / WITH A PARTNER

Differential equations and slope fields

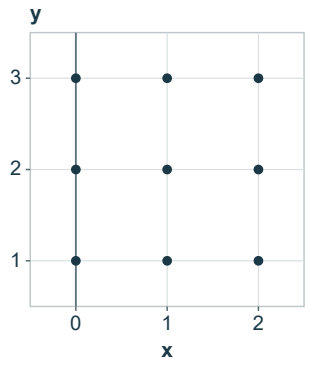
Work together on questions 1 and 2. Close the reference for question 3.

12.1 / TOGETHER

A culture begins with 80 cells. Its growth rate in cells per hour equals 0.15 times its current population. Write the initial-value problem, solve it, and give its growth rate when 200 cells are present.

12.2 / TOGETHER

For $y' = y - 2$, draw a short slope segment at each marked point with $x = 0, 1, 2$ and $y = 1, 2, 3$. On the same grid sketch the beginning of the solution through $(0, 3)$ until it leaves the window. Does it approach $y = 2$ as x increases? Explain.



12.3 / SOLO

Verify that $y = 1/(2 - x)$ solves $y' = y^2$, $y(0) = 1/2$. Give the largest interval containing the initial point on which this formula is a solution.

LESSON 12 / WORK INDEPENDENTLY

Differential equations and slope fields

Show your reasoning. Finish the exit problem before checking the key.

12.4 / SOLO

Solve $y' = 2xy$ with $y(1) = -3$. Identify any constant solution excluded by separation.

12.5 / MIXED

A drug amount satisfies $Q' = kQ$. Measurements are 24 mg at time zero and 18 mg after 2 hours. Find the signed rate constant k and the time until the amount reaches 12 mg.

12.6 / EXIT

For $y' = x(y - 1)$, a solution passes through $(0, 3)$. Predict the signs of its slope and concavity at that point, then solve the initial-value problem to check the prediction.

LESSON 13 / CED 8.1

The average height of a function

Reference topic 8.1 • CHA-4.B

8.1 / P1

Find the average value of $f(x) = 2 + \cos x$ on $[0, \pi/2]$ and use the range of f to check whether your answer is plausible.

8.1 / P2

A bin's mass is measured at times $t = 0, 2, 5$ minutes as $12, 18, 15$ kilograms. Use trapezoids to estimate its time-average mass. State why this is an estimate.

LESSON 13 / CED 8.2

Motion along a line

Reference topic 8.2 • CHA-4.C

8.2 / P1

A particle has velocity $v(t) = t^2 - 1$ m/s on $[0, 2]$ seconds. Find its displacement, total distance, and average velocity.

8.2 / P2

A cart begins at $s(0) = 7$ m with velocity $v(0) = 4$ m/s. Acceleration is -2 m/s^2 until $t = 5$ s. How far does it travel, and where does it finish? Explain the reversal.

LESSON 13 / CED 8.3

Accumulating a changing rate

Reference topic 8.3 • CHA-4.D / CHA-4.E

8.3 / P1

A parking lot begins with 70 cars. For the first 4 hours its arrival and departure rates are $20 - t$ and $8 + 2t$ cars per hour. Write a model for the count, interpret its integral, and find the final count.

8.3 / P2

A net flow sensor reads $3, 1, -3$ L/min at times $0, 2, 5$ min. Use trapezoids for an estimated net change and interpret its sign. If there are 8 L at minute 5, estimate the initial amount.

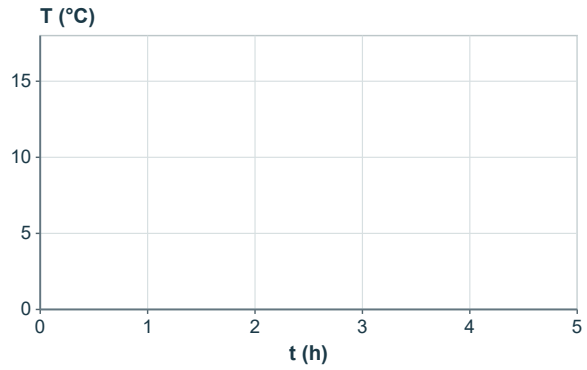
LESSON 13 / WITH A PARTNER

Net change, average value, and motion

Work together on questions 1 and 2. Close the reference for question 3.

13.1 / TOGETHER

A sensor trace joins $(0, 10)$, $(2, 16)$, and $(5, 10)$ with straight segments. The coordinates give time in hours and temperature in degrees Celsius. Find the average temperature over the five hours. Sketch the trace and the average-height rectangle.



13.2 / TOGETHER

At times 0, 1, and 3 h, a tank's inflow rates are 3, 5, and 4 L/h. Its outflow rates at the same times are 2, 2, and 3 L/h. Use a trapezoidal estimate to predict the amount at 3 h if it starts with 12 L.

13.3 / SOLO

A particle starts at position -2 m and has velocity $v(t) = 3 - 2t$ m/s on $0 \le t \le 4$ s. Find its final position and total distance. Show where you split your calculation.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

LESSON 13 / WORK INDEPENDENTLY

Net change, average value, and motion

Show your reasoning. Finish the exit problem before checking the key.

13.4 / SOLO

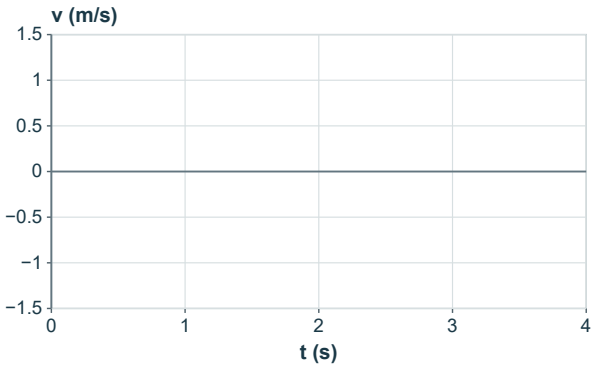
A cart has acceleration $a(t) = 6t - 6$ m/s^2 for $0 \le t \le 2$ s and initial velocity 3 m/s. Find its distance traveled and explain whether it reverses direction.

13.5 / MIXED

The delivery rate is $r(t) = 1 + t^2$ kilograms per hour on $[0, 3]$ h. Find the average rate and the time when the delivery rate equals that average. Why must at least one such time exist?

13.6 / EXIT

A cart starts at position 4 m. Its velocity varies linearly from 1 m/s at $t = 0$ to -1 m/s at $t = 2$, then linearly to 1 m/s at $t = 4$. Sketch its velocity graph. Find its final position, total distance, and average speed over these four seconds. Show how the signed contributions enter each calculation.



LESSON 14 / CED 8.4

Area from vertical strips

Reference topic 8.4 • CHA-5.A

8.4 / P1

Sketch $y = 3x$ and $y = x^2$ and mark a representative vertical strip. Calculate the area enclosed by the two curves.

8.4 / P2

On $[0, 1]$ find the area between $y = e^x - 2$ and $y = -2$. State what would happen to that area if both curves were raised by five units.

LESSON 14 / CED 8.5

Area from horizontal strips

Reference topic 8.5 • CHA-5.A

8.5 / P1

Sketch the region enclosed by $x = y^2$ and $x = 8 - y^2$. Mark a horizontal strip and find the area.

8.5 / P2

The region is bounded by $x = e^y$, the line $x = 1$, and the line $y = \ln 3$. Find its area with respect to y, stating how you found the lower limit.

LESSON 14 / CED 8.6

Area when curves change order

Reference topic 8.6 • CHA-5.A

8.6 / P1

The curves $y = \cos x$ and $y = \cos(2x)$ intersect several times on $[0, 2\pi]$. Use $\cos(2x) = 2\cos^2 x - 1$ to find every intersection. Determine which intersections require switching the upper curve, and calculate the total area between the curves.

8.6 / P2

A survey gives $\int_0^6 (f - g)\,dx = -2$. The graphs cross at $x = 0, 2, 4, 6$, and f is above g on the two outer intervals. Those outer areas are 3 and 4 square units. Find the total area between the curves.

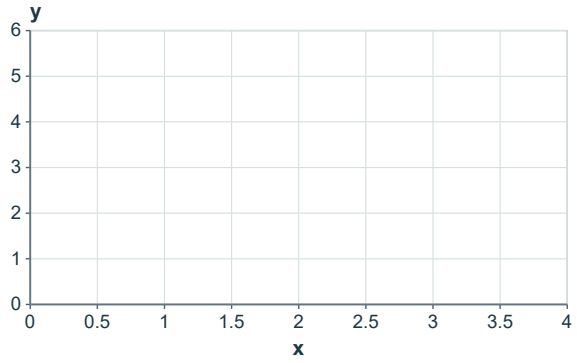
LESSON 14 / WITH A PARTNER

Area between curves

Work together on questions 1 and 2. Close the reference for question 3.

14.1 / TOGETHER

An upper boundary connects $(0,2)$, $(2,5)$, and $(4,2)$ by straight segments. The lower boundary is $y = 1$. Sketch the enclosed region from $x = 0$ to $x = 4$ and compute its area using the strip heights at the three labeled positions.



14.2 / TOGETHER

Find the area enclosed by $y = x$ and $y = x^2$. Identify the intersections and explain how you chose the upper boundary.

14.3 / SOLO

Find the area between $y = \sin x$ and $y = \sin(2x)$ on $[0, \pi]$. Use $\sin(2x) = 2 \sin x \cos x$ to locate intersections.

LESSON 14 / WORK INDEPENDENTLY

Area between curves

Show your reasoning. Finish the exit problem before checking the key.

14.4 / SOLO

The curves $x = y^2$ and $x = 4 - y^2$ enclose a region. Use one horizontal-strip integral to find its area and identify the widest strip.

14.5 / MIXED

Calculator allowed. Find the area between $y = e^{-x}$ and $y = x$ on $[0, 1]$ to three decimal places. Write the equations for the crossing and the area before evaluating.

14.6 / EXIT

Two continuous curves cross once inside $[0, 5]$, at $x = 2$. The graph of f is above g before the crossing. The area there is 3 square units, and $\int_0^5 (f - g)\, dx = -2$. Find the total area between the curves and justify the sign used on each interval.

LESSON 15 / CED 8.7

Square and rectangular cross sections

Reference topic 8.7 • CHA-5.B

8.7 / P1

For $0 \le x \le 2$ a base extends from $y = -x$ to $y = x$. Find the volume if slices perpendicular to the x-axis are squares.

8.7 / P2

A solid has rectangular sections perpendicular to the y-axis. At $0 \le y \le 2$, the side in the base runs from $x = y$ to $x = 4 - y$, and the section height is $1 + y$. Sketch a typical strip and evaluate the volume.

LESSON 15 / CED 8.8

Triangular and semicircular cross sections

Reference topic 8.8 • CHA-5.B

8.8 / P1

For $0 \le x \le 2$, the segment from $y = 0$ to $y = 2 - x$ is one leg of a right isosceles triangular cross section. Find the volume. How would it change if the segment were the hypotenuse?

8.8 / P2

A base extends from $x = 0$ to $x = 2y$ for $0 \le y \le 3$. Horizontal segments are the diameters of semicircles. Sketch the segment and its associated semicircle, then calculate the volume.

LESSON 15 / CED 8.9

Disks about a coordinate axis

Reference topic 8.9 • CHA-5.C

8.9 / P1

Rotate the region between $y = x^2$ and $y = 0$ on $[0, 2]$ about the x-axis. Find the volume and identify the step at which the fourth power arises.

8.9 / P2

The triangle bounded by $x = 0$, $y = 0$, and $x + 2y = 6$ rotates around the y-axis. Sketch a horizontal strip, calculate the volume, and check with the cone formula.

LESSON 15 / CED 8.10

Disks about a shifted axis

Reference topic 8.10 • CHA-5.C

8.10 / P1

Rotate the region between $y = 2$ and $y = 2 + \sin x$ for $0 \le x \le \pi$ about $y = 2$. Find the volume using $\sin^2 x = (1 - \cos(2x))/2$.

8.10 / P2

For $0 \le y \le 1$, rotate the strip region $y \le x \le 3$ about $x = 2$. Explain why every slice is a disk. Identify the farther endpoint and find the volume.

LESSON 15 / CED 8.11

Washers about a coordinate axis

Reference topic 8.11 • CHA-5.C

8.11 / P1

The region between $y = 3$ and $y = 1 + x$ on $0 \le x \le 2$ rotates around the x-axis. Sketch the region and label both radii on a strip. Find the volume.

8.11 / P2

For $0 \le y \le 2$, rotate the region $1 \le x \le 1 + y^2$ about the y-axis. Evaluate the washer integral.

LESSON 15 / CED 8.12

Washers about a shifted axis

Reference topic 8.12 • CHA-5.C

8.12 / P1

For $0 \le x \le 1$, rotate the region between $y = 0$ and $y = x$ about $y = 3$. Sketch a vertical slice and evaluate its volume integral.

8.12 / P2

For $0 \le x \le 3$, rotate the region $0 \le y \le x$ around $y = 1$. Write the integral in pieces and find its volume. Identify the slice or radius change at each break.

LESSON 15 / WITH A PARTNER

Volumes by slicing

Work together on questions 1 and 2. Close the reference for question 3.

15.1 / TOGETHER

A base is bounded by $y = 0$, $x = 0$, and $y = 4 - x$. Rectangles perpendicular to the x-axis have one side in the base and height twice that side. Sketch a rectangle and find the volume.

15.2 / TOGETHER

Rotate the region below $y = \sqrt{x + 1}$ and above $y = 0$ from $x = 0$ to $x = 3$ about the x-axis. Describe one slice and calculate the volume.

15.3 / SOLO

The region enclosed by $x = y^2$ and $x = 2y$ rotates about the y-axis. Express both radii in terms of y and find the volume.

LESSON 15 / WORK INDEPENDENTLY

Volumes by slicing

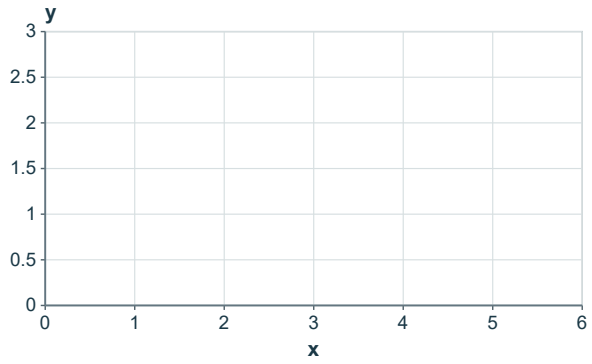
Show your reasoning. Finish the exit problem before checking the key.

15.4 / SOLO

Square slices of a solid are perpendicular to its length. At positions 0, 1, and 3 cm, their sides measure 4, 2, and 0 cm respectively. Apply the trapezoidal rule to the cross-sectional areas to estimate the volume.

15.5 / MIXED

For $0 \leq y \leq 1$, the region lies between $x = y^2$ and $x = 2 - y$. Rotate it about $x = 4$. Sketch a horizontal strip, label both radii, and find the volume.



15.6 / EXIT

For $0 \leq y \leq 3$, a region extends horizontally from $x = -2$ to $x = y$. Rotate it about the y-axis. A student uses washer area $\pi\left(y^2 - 4\right)$ throughout. Sketch a representative strip, explain the error, identify every input where the radius rule changes, and calculate the volume in cubic units.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 01 / Algebra, radians, and geometry

01A

Find the domain of $h(x) = \ln\left(9 - x^2\right)$.

01B

A mark on the rim of a wheel of radius 8 cm travels 10 cm along its circular path. Through what angle does the wheel turn, in radians and degrees?

01C / DELAYED

For fixed a and nonzero h, expand and simplify $\left[(a+2h)^2 - a^2\right]/h$. State the restriction and explain why it remains after cancellation.

Lesson 02 / Limits and continuity

02A

On $[1,3]$, $f(x) = 2x - 1$ for $x < 3$ and $f(3) = k$. Which k gives continuity at the right endpoint?

02B

Find $\lim_{x \to 0} x^4 \cos(3/x)$ without computing oscillating cosine values.

02C / DELAYED

For $g(x) = \frac{x+1}{x-2}$, state the limit as $x \to 2^-$ and the limit as $x \to +\infty$.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 03 / The meaning of a derivative

03A

A sensor reports $H(5.9) = 12.08$ cm and $H(6.1) = 12.22$ cm; time is in minutes. Estimate $H'(6)$.

03B

For $f(x) = x^3$, find $f'(-2)$ from the difference quotient. Show the simplification before taking the limit.

03C / DELAYED

A continuous graph meets at $(2, 5)$ with incoming slope 3 and outgoing slope 1. Does $f'(2)$ exist? Explain.

Lesson 04 / Basic differentiation rules

04A

Differentiate $f(x) = x^{-2}\cos x$ on its domain.

04B

Find the slope of $y = \frac{\ln x}{x}$ at $x = e$.

04C / DELAYED

A differentiable g satisfies $g(1) = -2$ and $g'(1) = 5$. For $p(x) = (x + 3)\,g(x)$, find $p'(1)$.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 05 / Composite, implicit, and inverse functions

05A

Find $f'''(x)$ if $f(x) = \cos(2x)$.

05B

A locally invertible differentiable function has $f(-1) = 7$ and $f'(-1) = -2/3$. Find the derivative of its inverse at 7.

05C / DELAYED

Differentiate $y = \arcsin(x/4)$ and give the interval on which your derivative is valid.

Lesson 06 / Related rates and local models

06A

A cube's side is 3 cm and shrinking at 0.2 cm/s. At what rate is its volume changing?

06B

Use the tangent line to $\ln x$ at $x = 1$ to estimate $\ln(1.08)$. Is the estimate high or low?

06C / DELAYED

Find k such that $\lim_{x \to 0} \frac{e^{kx} - 1}{\sin(3x)} = 2$. Identify the form before applying a rule.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 07 / Using derivatives to analyze graphs

07A

Suppose $f'(x) = (x-1)^2(x+2)$ everywhere. Which stationary point is a local extremum? Explain the sign change.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

07B

If g is continuous on $[2,8]$, differentiable on $(2,8)$, and $g(2) = 9$, $g(8) = 6$, what slope must occur inside the interval?

07C / DELAYED

The graph of f' is rising on $(0,2)$ and falling on $(2,5)$; f is differentiable throughout $(0,5)$. What changes at $x = 2$?

PRACTICE BANK / CONTINUED

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 08 / Optimization and implicit curves

08A

Two positive numbers sum to 18. For numbers x and $18 - x$, maximize $x\,(\,18 - x\,)^{\,2}$. Which value of x gives the maximum?

08B

On $xy + y^2 = 6$, find the tangent slope at $(\,1, 2\,)$. Is the local branch rising or falling there?

08C / DELAYED

A quantity $Q\,(\,x\,) = x^2 - 6x + 14$ can be chosen only for $4 \le x \le 8$. Where is its minimum? Justify the endpoint.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 09 / Approximating a definite integral

09A

Write $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{3}{n}\sqrt{1+\frac{3k}{n}}$ as a definite integral in x using $[1,4]$.

09B

Given $f(0) = 1, f(1) = 5, f(4) = 3$, estimate $\int_0^4 f(x)\,dx$ with two trapezoids.

09C / DELAYED

If $\int_2^5 f(x)\,dx = -4$ and $\int_2^8 f(x)\,dx = 6$, evaluate $\int_8^5 f(x)\,dx$.

Lesson 10 / The Fundamental Theorem of Calculus

10A

If $H(x) = \int_x^{2x} \sin\left(t^2\right)\,dt$, find $H'(x)$.

10B

For $A(x) = \int_1^x f(t)\,dt$, continuous f crosses from negative to positive at $x = 4$. What does A do there?

10C / DELAYED

Continuous r has antiderivative F with $F(1) = -2$ and $F(4) = -7$. Find $\int_1^4 3r(x)\,dx$.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 11 / Antiderivatives and substitution

11A

Evaluate $\int \frac{1}{x^2 - 6x + 13}\,dx$. Complete the square first.

11B

Integrate $\frac{x^2 + 2x + 3}{x + 1}$ on an interval avoiding $x = -1$.

11C / DELAYED

Use substitution to evaluate $\int_0^2 x e^{x^2 + 1}\,dx$.

Lesson 12 / Differential equations

12A

For $y' = 2x - y$, what slope is drawn at $(1, 2)$? Is $y = 2x$ itself a solution?

12B

Verify that $y = 3 - 5e^{-t}$ solves $dy/dt = 3 - y$. What initial condition at $t = 0$ does it satisfy?

12C / DELAYED

For $y' = x(y - 4)$ and $y(0) = 4$, give a solution. Why could separation overlook it?

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 13 / Rate, motion, and accumulation

13A

A particle has velocity $v(t) = 2t - 4$ m/s on $[0, 3]$. Find displacement and total distance.

13B

A motor draws power $P(t) = 6e^{-t/2}$ kW during its first two hours, where t is measured in hours. Find its average power.

13C / DELAYED

A bin starts with 24 kg. Material enters at 5 kg/min and leaves at $2 + t$ kg/min for $0 \le t \le 4$. How much remains at 4 minutes?

Lesson 14 / Area between curves

14A

Find the area bounded by $x = y^2$ and $x = 4$. Integrate with respect to y.

14B

Find the total area enclosed by the curves $y = x$ and $y = x + (x + 1)x(x - 2)$ between their leftmost and rightmost intersections. Use a separate sheet for the intersection ordering, signed integrals, and evaluation.

14C / DELAYED

A horizontal slice of a region extends from $x = \sin y$ to $x = 2 + \cos y$ for $0 \le y \le \pi/2$. Set up and evaluate its area.

PRACTICE BANK

Fluency and retrieval

Work A and B after instruction. Save C for a different day. Use blank paper if your work needs more room.

Lesson 15 / Volumes by slicing

15A

Rotate $0 \le y \le x$, $0 \le x \le 2$, about $y = -2$. Find the volume using washers.

15B

Cross sections perpendicular to the x-axis are right isosceles triangles. The segment of length $2 + x$ across the base is the hypotenuse, for $0 \le x \le 2$. Find the volume.

15C / DELAYED

Rotate the region $y \le x \le 2y$, $0 \le y \le 1$, about $x = 5$. Which boundary gives the outer radius? Find the volume.

CHECKPOINT 1

Differential calculus

After lessons 01–08, allow 30–35 minutes. Work independently without notes or a calculator. Give the reason for each conclusion. Keep partial work so you can identify the first step that needs repair.

QUESTION 1 / 3 POINTS

For $f(x) = x^2 + 2x$, use the limit of a difference quotient to find $f'(1)$. Give the tangent line at $x = 1$.

QUESTION 2 / 3 POINTS

A differentiable function has derivative $f'(x) = (x + 1)(x - 2)^2$. At each critical number, classify the behavior of f and justify it using signs.

Mark the inputs where a sign can change. Label each row with the expression you are testing.

QUESTION 3 / 3 POINTS

For $g(x) = \sqrt{x + 1}$ on $[0, 3]$, explain why the Mean Value Theorem applies. Find every point promised by its conclusion on this interval.

CHECKPOINT 1

Differential calculus

Continued. Keep your reasoning visible.

QUESTION 4 / 3 POINTS

The circumference of a circular ripple increases at 2π cm/s. When its radius is 4 cm, how fast does its enclosed area change? Show the time derivatives.

QUESTION 5 / 3 POINTS

A rectangular pen must enclose 72 square meters. Fencing for two opposite sides costs 4 dollars per meter; fencing for the other two costs 2 dollars per meter. Choose the dimensions that minimize cost and justify your choice.

CHECKPOINT 2

Accumulation and change

After lessons 09–12, allow 30–35 minutes without notes or a calculator. This check also revisits earlier differentiation. Keep units attached to rates and accumulated quantities.

QUESTION 1 / 3 POINTS

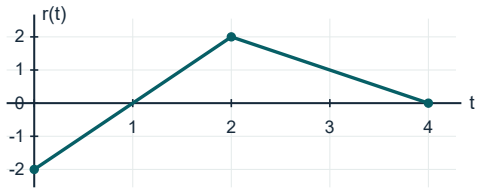
Use the tangent line to $f(x) = 10/x$ at $x = 4$ to estimate $f(4.08)$. Determine whether the estimate is high or low and justify your answer.

QUESTION 2 / 3 POINTS

An inflow rate is strictly concave down from minute 0 to minute 4. At times $t = 0, 1, 3, 4$ minutes, the rates are $2, 5, 8, 9$ L/min. Use a trapezoidal sum to estimate the amount added. Decide whether the estimate is high or low and explain how you know.

QUESTION 3 / 3 POINTS

The graph below shows r. Let $A(x) = \int_0^x r(t)\,dt$. Find the absolute minimum value of A on $[0, 4]$ and evaluate $A(4)$.



CHECKPOINT 2

Accumulation and change

Continued. Keep your reasoning visible.

QUESTION 4 / 3 POINTS

Evaluate $\int_0^2 \frac{6x}{3x^2+2}\,dx$ exactly. Show how your choice of variable changes both integration limits.

QUESTION 5 / 3 POINTS

A function satisfies $\frac{dy}{dx} = 2x\,(\,1+y\,)$ and $y\,(\,0\,) = 2$. Solve the initial-value problem and identify a constant solution of the differential equation that separation would exclude.

FINAL READINESS A

Without a calculator

After lesson 15, take 25 minutes on paper. You may take Part B another day. Write enough reasoning for someone to check your method; use exact values. These are home mastery tasks.

QUESTION 1 / 3 POINTS

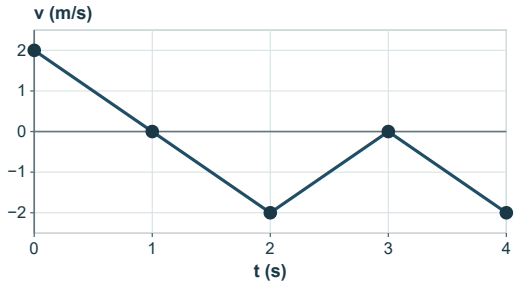
For $x \le -1$, let $f(x) = x^2 + 1$. For $x > -1$, let $f(x) = ax + 3$, where a is a constant. Can one value of a make f both continuous and differentiable at -1? Justify your conclusion using one-sided limits and slopes.

QUESTION 2 / 3 POINTS

The curve $x^2y + y^2 = 5$ passes through $(2, 1)$. Find the slope there, then estimate y on the same branch when $x = 2.06$.

QUESTION 3 / 3 POINTS

A cart moves on a straight track. The graph below gives its velocity in meters per second, with straight segments joining the vertices. Find its displacement and total distance from $t = 0$ to $t = 4$ seconds. At which times during the trip does it reverse direction? Justify your answers from the graph.



FINAL READINESS A

Without a calculator

Continued. Keep your reasoning visible.

QUESTION 4 / 3 POINTS

Solve $\frac{dy}{dt} = \frac{t}{y}$ with $y(0) = 2$. Check the differential equation and the initial condition in your answer.

QUESTION 5 / 3 POINTS

A solid has base $0 \le x \le 4$ and $0 \le y \le \sqrt{x}$ in the xy-plane. Cross sections perpendicular to the x-axis are semicircles whose diameters span the base. Find the base area and the solid's volume.

FINAL READINESS B

With a calculator

Allow 25 minutes with a graphing calculator in radian mode. Show the equation or definite integral before using the numerical feature. Carry extra digits during work and report final numerical values to three decimal places.

QUESTION 1 / 3 POINTS

A tank starts with 40 L. During $0 \le t \le 6$ minutes, water flows in at $6 + 3 \sin(t/2)$ L/min and out at 7 L/min. Find the amount of water at $t = 6$ and the average net rate of change over the interval.

QUESTION 2 / 3 POINTS

For the same tank and the rates given in Question 1, use a graph or numerical solver to determine when the amount of water is greatest on $0 \le t \le 6$. Give that maximum amount and justify the global comparison.

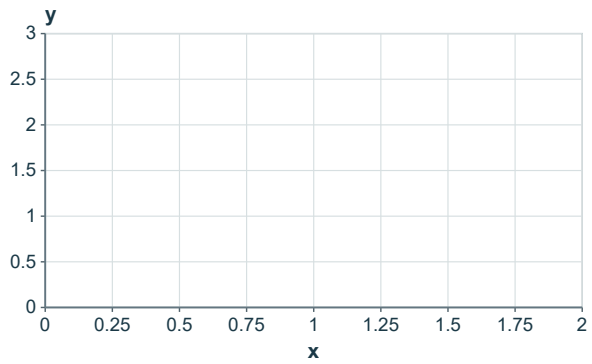
FINAL READINESS B

With a calculator

Continued. Keep your reasoning visible.

QUESTION 3 / 3 POINTS

In the first quadrant, a region is bounded by $x = 0$, $y = 2\cos x$, and $y = x^2$, up to the first positive intersection. Graph the curves, find that intersection's x-coordinate, and calculate the area.



QUESTION 4 / 3 POINTS

The region between $y = 2\cos x$ and $y = x^2$ for $0 \le x \le c$, where c is their first positive intersection, rotates around $y = -1$. Write a washer integral and evaluate the volume.

DELAYED PROBES A

AB units 1–4

Use one fresh item 2–7 days after the relevant repair. Start from blank paper without hints or notes. Explain your first decision. A prompt-assisted response calls for a different delayed problem before recording independent mastery.

QUESTION 1 / 3 POINTS

For $0 < |x - 1| < 1$, an unknown function satisfies $2 - (x - 1)^2 \leq F(x) \leq 2 + 2|x - 1|$. Find $\lim_{x \to 1} F(x)$, justify your conclusion, and determine the value of $F(1)$ that would ensure continuity.

QUESTION 2 / 3 POINTS

At $t = 3.9, 4.0, 4.1$ minutes, measured temperatures T were $10.4, 10.0, 9.8$ degrees Celsius, respectively. Estimate $T'(4)$ using the observations on both sides. Explain the result and one limitation.

DELAYED PROBES A

AB units 1–4

Continued. Keep your reasoning visible.

QUESTION 3 / 3 POINTS

The differentiable function g is strictly increasing and $g(3) = 5$, $g'(3) = 2$. Find $\left(g^{-1}\right)'(5)$ and $\frac{d}{dx}\left[g\left(x^2\right)\right]$ at $x = \sqrt{3}$.

QUESTION 4 / 3 POINTS

At 2 seconds, the radius of a sphere is 3 cm and increases at 0.2 cm/s. Find its instantaneous volume rate; use it to estimate the volume increase during the next 0.1 second. Use $V = \frac{4}{3}\pi r^3$.

DELAYED PROBES B

AB units 5–8

Use one fresh item from the relevant unit on a later day. Work without notes or a calculator. If an answer is incomplete, use its lesson route, then select a different problem with the same reasoning for a subsequent independent check.

QUESTION 1 / 3 POINTS

The graph of f' consists of line segments joining $(-2,1)$, $(0,-1)$, $(2,1)$ in order. Assume f is differentiable on $(-2,2)$. Give the x-coordinates of the local extrema, classify each, and give the x-coordinate of every inflection point. Justify each conclusion.

QUESTION 2 / 3 POINTS

Evaluate $\int_0^1 \frac{x^2}{x+1}\,dx$ exactly. Use algebra to rewrite the integrand, then check your antiderivative by differentiating.

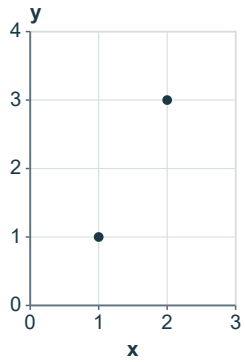
DELAYED PROBES B

AB units 5–8

Continued. Keep your reasoning visible.

QUESTION 3 / 3 POINTS

For the slope field of $dy/dx = y - x^2$, sketch short segments at $(1,1)$ and $(2,3)$. A student says $y = x^2$ is a solution because the field is horizontal along that parabola. Test the claim.



QUESTION 4 / 3 POINTS

The curves $x = y^2 - 1$ and $x = 2y + 2$ enclose a finite region. Find its area with a single integral in y. Justify your bounds and the ordering of the curves.

OPTIONAL CUMULATIVE CASE A

Two runners on a straight path

Calculator allowed, in radians. Two runners move in the positive direction. At $t = 0$, runner A is 2 meters behind runner B. For $0 \le t \le 10$ seconds their velocities in m/s are $v_A(t) = 6 + 0.4t + 1.2\sin\left(t/2\right)$ and $v_B(t) = 7 + 0.15t$. Define the signed lead $G(t) = s_A(t) - s_B(t)$. A positive lead places A ahead. Show each equation or integral before using a numerical feature. Round final numerical answers to three decimals.

CASE A / PART 1

On your graphing calculator, graph both velocities and find every time they agree. Use the sign of their difference on either side to explain what happens to the signed lead.

CASE A / PART 2

Find $G(6)$. Identify the leading runner at 6 seconds and interpret your value in meters.

OPTIONAL CUMULATIVE CASE A

Two runners on a straight path

Calculator allowed, in radians. Two runners move in the positive direction. At $t = 0$, runner A is 2 meters behind runner B. For $0 \le t \le 10$ seconds their velocities in m/s are $v_A(t) = 6 + 0.4t + 1.2\sin\left(t/2\right)$ and $v_B(t) = 7 + 0.15t$. Define the signed lead $G(t) = s_A(t) - s_B(t)$. A positive lead places A ahead. Show each equation or integral before using a numerical feature. Round final numerical answers to three decimals.

CASE A / PART 3

At 6 seconds, does A's lead increase or decrease? Is the rate at which the lead changes itself increasing or decreasing? Give both rates and their units.

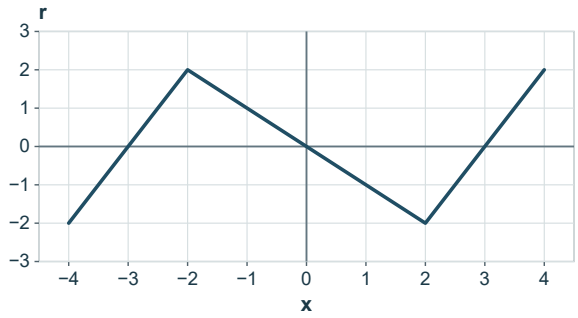
CASE A / PART 4

When does A first draw level with B? Explain why this is the first such time.

OPTIONAL CUMULATIVE CASE B

Reading an accumulation function from its rate graph

No calculator. The graph of the continuous function r consists of line segments connecting $(-4,-2)$, $(-2,2)$, $(2,-2)$, $(4,2)$ in order. Define $H(x) = 1 + \int_0^x r(t)\,dt$ for $-4 \le x \le 4$. Label signed areas and use derivative evidence in your justifications.



CASE B / PART 1

Calculate $H(-4)$ and $H(4)$. Show how reversing the integration limits affects the left-hand calculation.

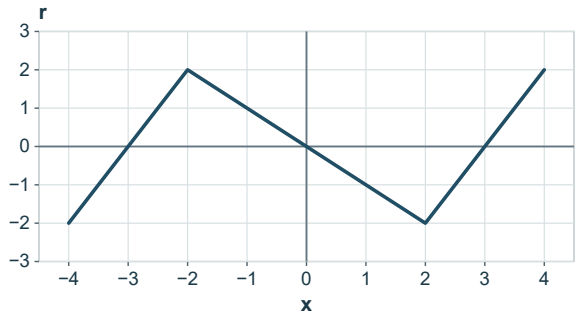
CASE B / PART 2

Find the absolute maximum and minimum values of H and every input attaining either one.

OPTIONAL CUMULATIVE CASE B

Reading an accumulation function from its rate graph

No calculator. The graph of the continuous function r consists of line segments connecting $(-4,-2)$, $(-2,2)$, $(2,-2)$, $(4,2)$ in order. Define $H(x) = 1 + \int_0^x r(t)\,dt$ for $-4 \le x \le 4$. Label signed areas and use derivative evidence in your justifications.



CASE B / PART 3

Find every inflection input for H. A student claims $x = 0$ is another one because $r(0) = 0$. Evaluate the claim.

CASE B / PART 4

Let $K(x) = H(2x - 1)$. Use the tangent to K at $x = 1$ to approximate $K\left(26/25\right)$. Determine whether your estimate is high or low.